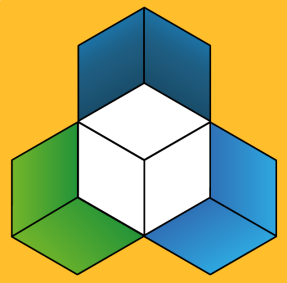


The Complete Mathematics Conference

Nathan Day

Daydream Interweaver

Reciprocals

Some Maths

Write $15 : 6$ in the form:

a) $1 : n$ → $1 : \frac{2}{5}$

b) $n : 1$ → $\frac{5}{2} : 1$

Solve:

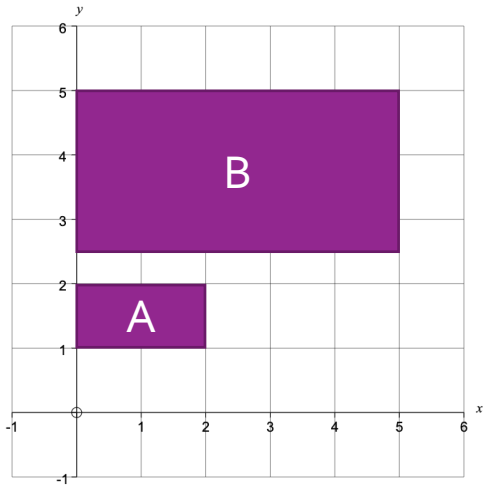
$$10x^2 - 29x + 10 = 0$$

$x = \frac{5}{2}, x = \frac{2}{5}$

I walk 15 miles in 6 hours.

a) How long does it take me to walk a mile? → $\frac{2}{5}$ of an hour

b) How many miles do I walk in one hour? → $\frac{5}{2}$ miles

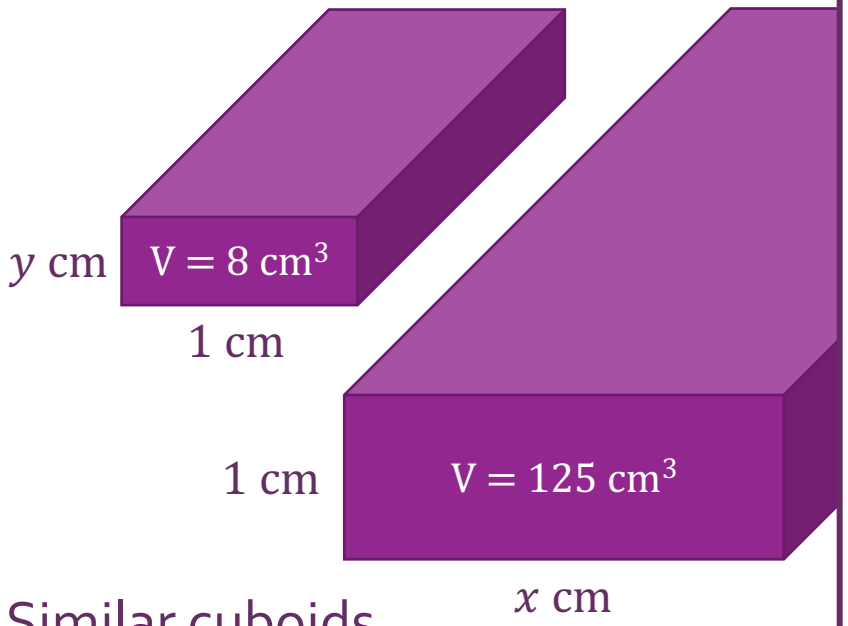


a) Transformation from A to B?

Enlargement s.f. $\frac{5}{2}$ about O

b) Transformation from B to A?

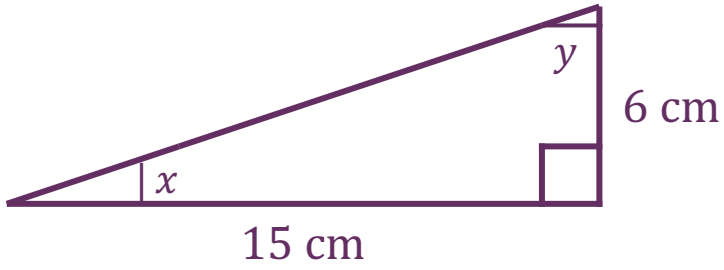
Enlargement s.f. $\frac{2}{5}$ about O



Similar cuboids.

a) $x =$ → $\frac{5}{2}$ cm

b) $y =$ → $\frac{2}{5}$ cm



a) $\tan(x) =$ → $\frac{2}{5}$

b) $\tan(y) =$ → $\frac{5}{2}$

Interweaving

Using questions and tasks that bring together multiple different topics from across mathematics.

Interweaving

It’s not interleaving!

Why do Interweaving?

1 – Connections

2 – Retrieval

3 – Depth

4 – Challenge

5 – Purpose

Reciprocals

Not ‘flipping the fraction’...

Not ‘1 over the thing’...

The multiplicative inverse!
(What you need to multiply by to get 1)

Reciprocals with… Decimals and Mixed Numbers

Find the reciprocal of each of the following, leaving your answers as simplified fractions:

a) $1\frac{1}{2}$

b) $-3\frac{5}{8}$

c) $13\frac{1}{2}+21\frac{1}{3}$

d) $34-55\frac{1}{4}$

e) $\frac{8}{9}\times1\frac{1}{44}$

f) 0.5

g) $0.\dot{3}$

h) $0.9\dot{8}$

i) $-1.\dot{0}3\dot{7}$

j) $1.\dot{0}0\dot{1}$

Reciprocals with… Decimals and Mixed Numbers

Find the reciprocal of each of the following, leaving your answers as simplified fractions:

a) $1\frac{1}{2}$ $\rightarrow \frac{2}{3}$

b) $-3\frac{5}{8}$ $\rightarrow -\frac{8}{29}$

c) $13\frac{1}{2}+21\frac{1}{3}$ $\rightarrow \frac{6}{209}$

d) $34-55\frac{1}{4}$ $\rightarrow -\frac{4}{85}$

e) $\frac{8}{9}\times 1\frac{1}{44}$ $\rightarrow \frac{11}{10}$

f) 0.5 $\rightarrow 2$

g) $0.\dot{3}$ $\rightarrow 3$

h) $0.9\dot{8}$ $\rightarrow \frac{90}{89}$

i) $-1.\dot{0}3\dot{7}$ $\rightarrow \frac{27}{28}$

j) $1.\dot{0}0\dot{1}$ $\rightarrow \frac{999}{1000}$

Interweaving **in**

Reciprocals with… Indices

Interweaving **in**

Reciprocals with… Indices

1) Find the reciprocal of each of the following, leaving your answers in index form:

a) $3^4 \rightarrow 3^{-4}$

b) $5^{-6} \rightarrow 5^6$

c) $7^{\frac{8}{9}} \rightarrow 7^{-\frac{8}{9}}$

d) $x^{10} \rightarrow x^{-10}$

e) $11 \times 11 \times 11 \rightarrow 11^{-3}$

f) $12^{13} \times 12^{14} \rightarrow 12^{-27}$

g) $15^{16} \div 15^{17} \rightarrow 15$

h) $(18^{-19})^{20} \rightarrow 18^{380}$

i) $(21xy^2)^{-2} \rightarrow 21^2x^2y^4$

j) $\frac{2^3}{2^4+2^5} \rightarrow 6$

2) Solve the following:

e.g. $x^{\frac{2}{3}} = 9$

$\Rightarrow \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 9^{\frac{3}{2}}$

$\Rightarrow x = 27$

a) $x^{\frac{4}{5}} = 16 \rightarrow x = 32$

b) $x^{-\frac{6}{7}} = 1\,000\,000 \rightarrow x = 10\,000\,000$

Interweaving **in**

Reciprocals with… Standard Form

1) Find the reciprocal of each of the following, leaving your answers in standard form:

a) 1×10^5

b) 5×10^{-4}

c) 2.5×10^3

d) 1.25×10^{-2}

e) 6.25×10^1

Interweaving **in**

Reciprocals with… Standard Form

1) Find the reciprocal of each of the following, leaving your answers in standard form:

a) $1 \times 10^{5} \rightarrow 1 \times 10^{-5}$

b) $5 \times 10^{-4} \rightarrow 2 \times 10^{3}$

c) $2.5 \times 10^{3} \rightarrow 4 \times 10^{-4}$

d) $1.25 \times 10^{-2} \rightarrow 8 \times 10^{1}$

e) $6.25 \times 10^{1} \rightarrow 1.6 \times 10^{-2}$

2) Using your answers to (1), solve each of the following:

a) $2500x = 7 \rightarrow x = 2.8 \times 10^{-3}$

b) $0.0005x = 8 \rightarrow x = 1.6 \times 10^{4}$

c) $62.5x = 9 \rightarrow x = 1.44 \times 10^{-1}$

d) $\frac{0.0125x}{100000} = 10 \rightarrow x = 8 \times 10^{7}$

Interweaving **in**

Reciprocals with… Surds

Find the reciprocal of each of the following, leaving your answers as simplified surds:

a) $\sqrt{3}$

b) $\frac{\sqrt{2}}{2}$

c) $\frac{\sqrt{5}}{15}$

d) $\frac{\sqrt{5}+1}{2}$

e) $\sqrt{2}+1$

Reciprocals with… Surds

Find the reciprocal of each of the following, leaving your answers as simplified surds:

a) $\sqrt{3}$ $\rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $\frac{\sqrt{2}}{2}$ $\rightarrow \frac{2}{\sqrt{2}} = \sqrt{2}$

c) $\frac{\sqrt{5}}{15}$ $\rightarrow \frac{15}{\sqrt{5}} = 3\sqrt{5}$

d) $\frac{\sqrt{5}+1}{2}$ $\rightarrow \frac{2}{\sqrt{5}+1} = \frac{2\sqrt{5}-2}{4} = \frac{\sqrt{5}-1}{2}$

e) $\sqrt{2}+1$ $\rightarrow \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$

Reciprocals with… Bounds

Find the upper and lower bounds of the reciprocals of each of the following as simplified fractions:

a) 3 (rounded to the nearest whole number)

b) 2.5 (rounded to 1 decimal place)

c) 2 (truncated to a whole number)

d) 1.50 (rounded to 3 significant figures)

e) 1 (rounded to 1 significant figure)

Interweaving **in**

Reciprocals with… Bounds

Find the upper and lower bounds of the reciprocals of each of the following as simplified fractions

a) 3 (rounded to the nearest whole number)	$\frac{2}{7}$	$\frac{2}{5}$
b) 2.5 (rounded to 1 decimal place)	$\frac{20}{51}$	$\frac{20}{49}$
c) 2 (truncated to a whole number)	$\frac{1}{3}$	$\frac{1}{2}$
d) 1.50 (rounded to 3 significant figures)	$\frac{200}{301}$	$\frac{200}{299}$
e) 1 (rounded to 1 significant figure)	$\frac{2}{3}$	$\frac{20}{19}$

Interweaving **in**

Reciprocals with… Equations

By forming and solving an equation, find the following

a) A number that is $\frac{1}{4}$ of its reciprocal

b) A number that is 36% of its reciprocal

c) Two numbers that are their own reciprocals

d) Two numbers that are 2.1 greater than their reciprocals

e) Two numbers that are 1 greater than their reciprocals

*f) A number and its reciprocal that have a mean of $\frac{29}{20}$

Interweaving **in**

Reciprocals with… Equations

By forming and solving an equation, find the following

a) A number that is $\frac{1}{4}$ of its reciprocal → $\frac{1}{2}$ (or $-\frac{1}{2}$)

b) A number that is 36% of its reciprocal → $\frac{3}{5}$ (or $-\frac{3}{5}$)

c) Two numbers that are their own reciprocals → 1 and -1

d) Two numbers that are 2.1 greater than their reciprocals → $-\frac{2}{5}$ and $\frac{5}{2}$

e) Two numbers that are 1 greater than their reciprocals → $\frac{1\pm\sqrt{5}}{2}$

*f) A number and its reciprocal that have a mean of $\frac{29}{20}$ → $\frac{2}{5}$ and $\frac{5}{2}$

But this isn't the only way to interweave reciprocals!

Instead of interweaving **in**...

we can interweave **out**!

Reciprocals with… The Harmonic Mean

I run to school at 6 mph, and then run back home at 4 mph.
What was my average speed?

Gallium has a density of $6\ \mathrm{g/cm^3}$.
Barium has a density of $4\ \mathrm{g/cm^3}$.
What is the density of an alloy of 240 g of each metal?

Anne takes 6 hours to paint a wall.
Bob takes 4 hours to paint wall.
How long does it take them to paint the wall, working together?

Substitute $x = 6$ and $y = 4$ into:

$$\frac{2}{\frac{1}{x} + \frac{1}{y}}$$

A 6 ohm resistor and a 4 ohm resistor are placed in a circuit in parallel. What is the overall resistance?

Whenever you have a multiplicative relationship, there are reciprocals lurking around somewhere.

Interweaving **out**

Write $15 : 6$ in the form:

a) $1 : n$

b) $n : 1$

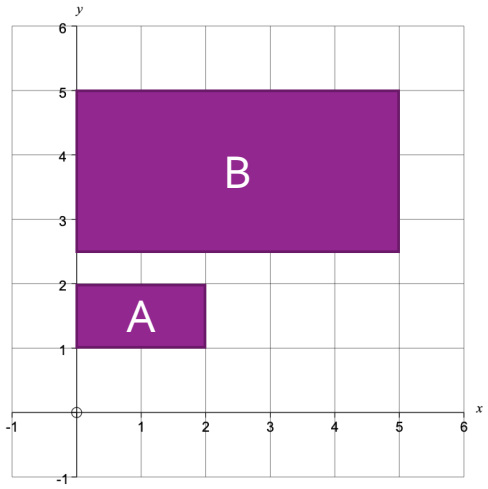
Solve:

$$10x^2 - 29x + 10 = 0$$

I walk 15 miles in 6 hours.

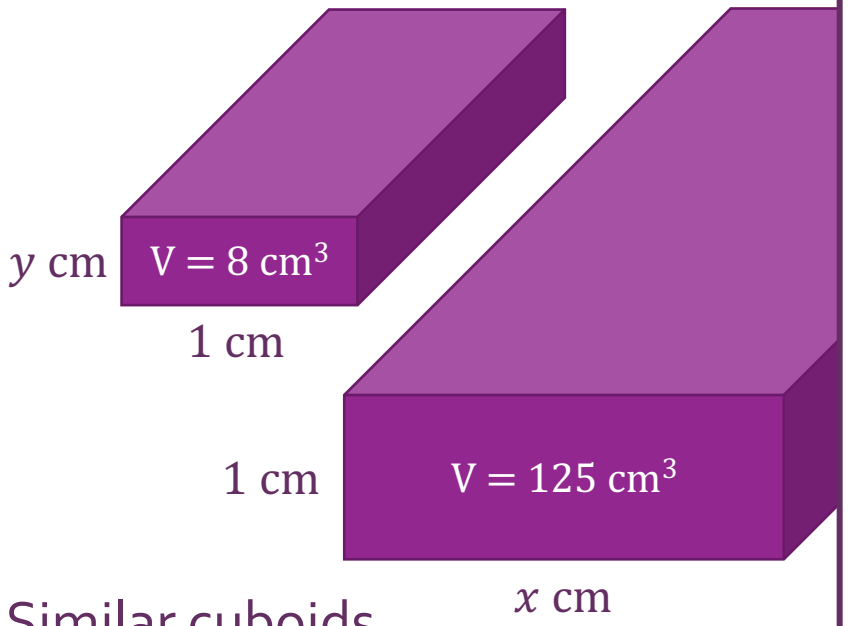
a) How long does it take me to walk a mile?

b) How many miles do I walk in one hour?



a) Transformation from A to B?

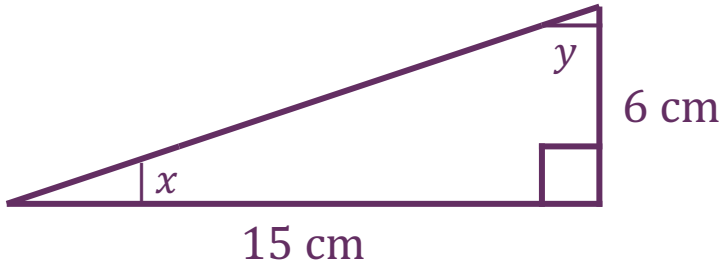
b) Transformation from B to A?



Similar cuboids.

a) $x =$

b) $y =$



a) $\tan(x) =$

b) $\tan(y) =$

Interweaving **out**

Write $15 : 6$ in the form:

a) $1 : n$

b) $n : 1$

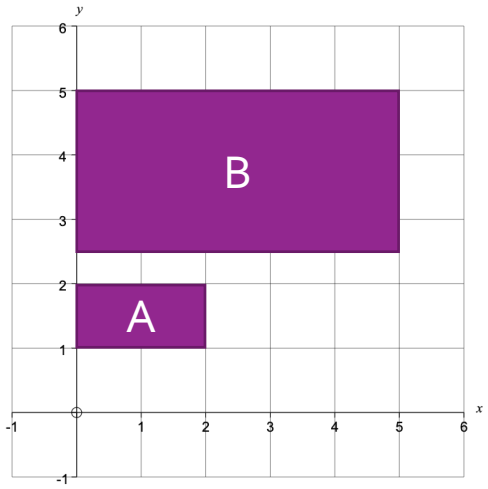
Solve:

$$10x^2 - 29x + 10 = 0$$

I walk 15 miles in 6 hours.

a) How long does it take me to walk a mile?

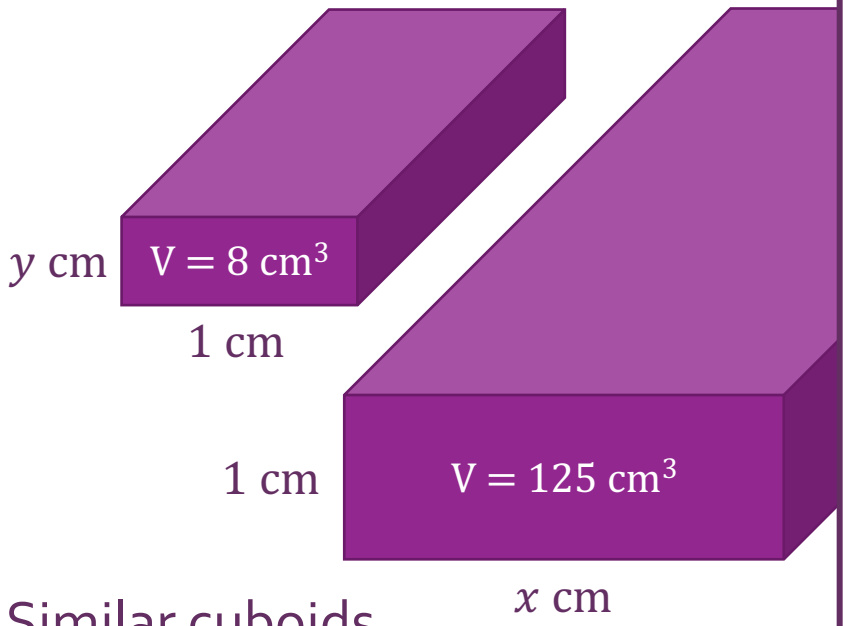
b) How many miles do I walk in one hour?



a) Transformation from A to B?

b) Transformation from B to A?

Reciprocals!



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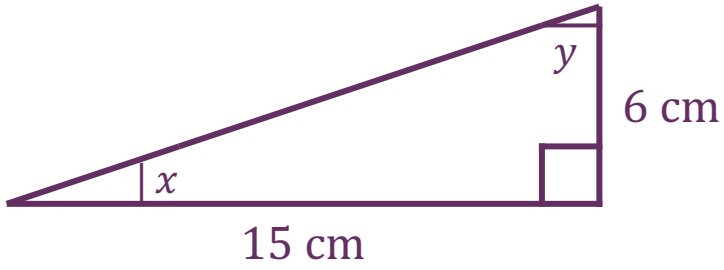


a) $\tan(x) =$

b) $\tan(y) =$

Interweaving **out**

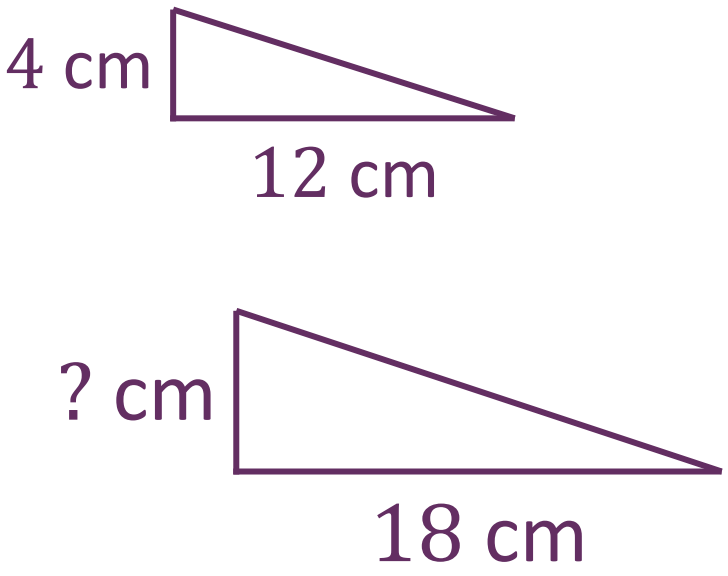
Maryse walks 12 miles in 4 hours.
How long does it take her to walk 18 miles?

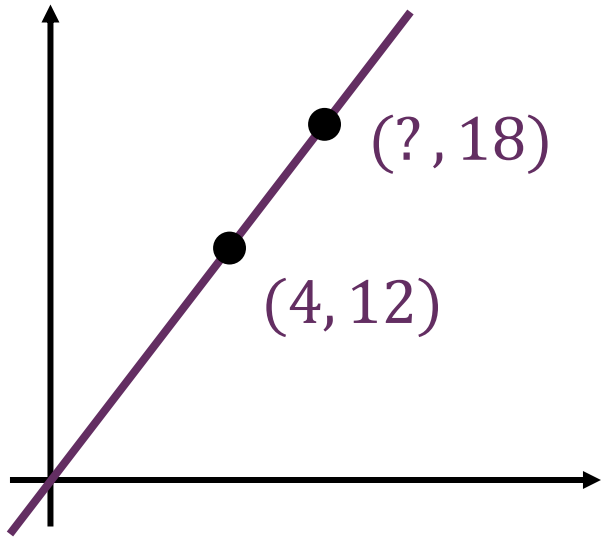
$$\frac{4}{12} = \frac{?}{18}$$

4 yards is equal to 12 feet.

How many yards are there in 18 feet?

Proportion!





Kathryn earns £4 for every 12 trees she plants.

How much does she earn for planting 18 trees?

Interweaving **out**

Nikki spilled some ink over her maths homework.

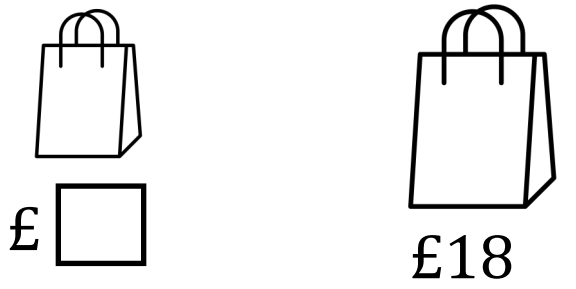
Can you still answer the question?

$\square(12x + 4) = \square(18x + \square)$

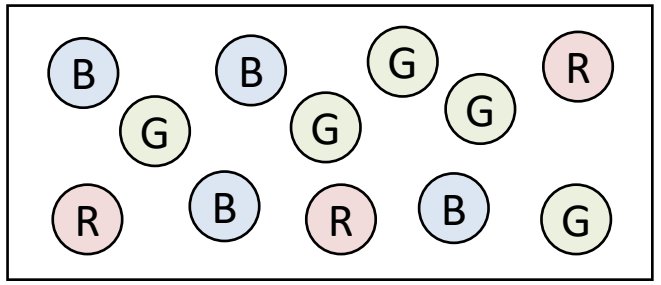
A rectangle has perimeter 12 cm and width 4 cm.

An enlargement of that rectangle has perimeter 18 cm. What is its width?

David and Amber share money in the ratio $4 : 12$.



Proportion!



Charlotte takes a counter from the box at random 18 times. Estimate the number of times she will take a blue counter.

$$12x = 4y$$

y is equal to 18. What is the value of x?

12% of a number is 4.

What is 18% of that number?

Interweaving **out**

Complete the square:

$$x^2 + 8x + 40$$

I have 40 metres of ribbon.
I cut off enough ribbon to make a square with side length 4 m.
How much is left?

Find the nth term:

$$28, 32, 36, 40, \ldots$$

Corrections?

Find the equation of the line parallel to $y = 4x - 7$ that passes through $(4, 40)$

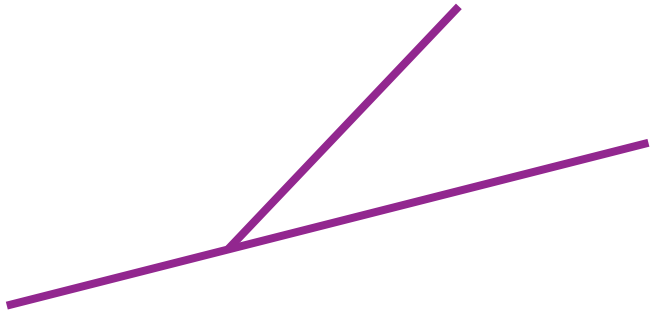
Solve the simultaneous equations:

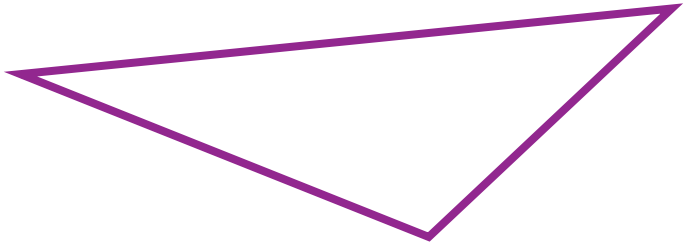
$$4x + 2y = 64$$
$$4x + \ y = 40$$

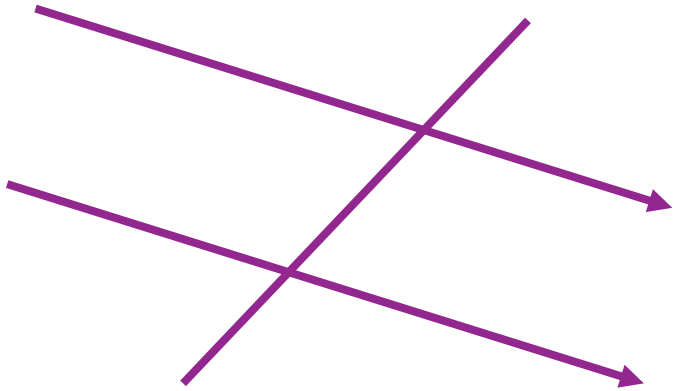
Anne and Bob share money in the ratio $2 : 6$.
Bob gets £16 more than Anne.

How much does Bob get?

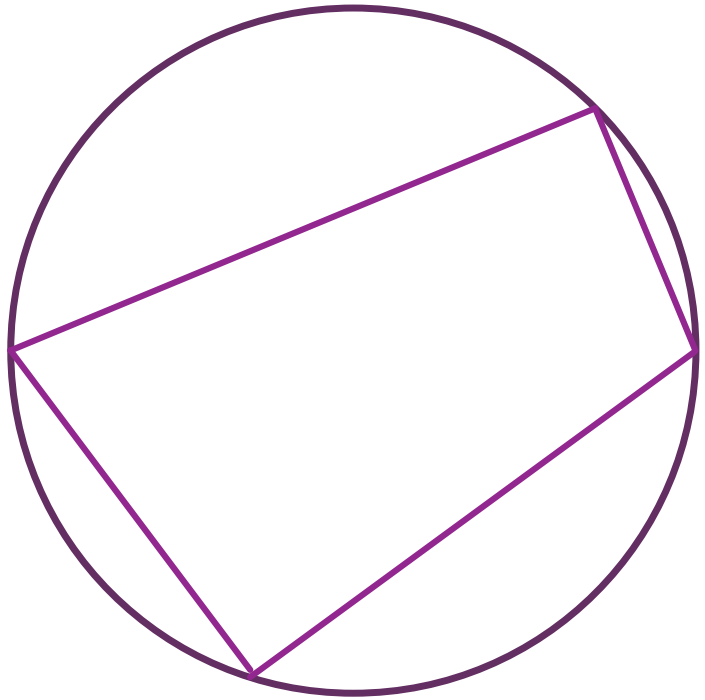
Interweaving **out**

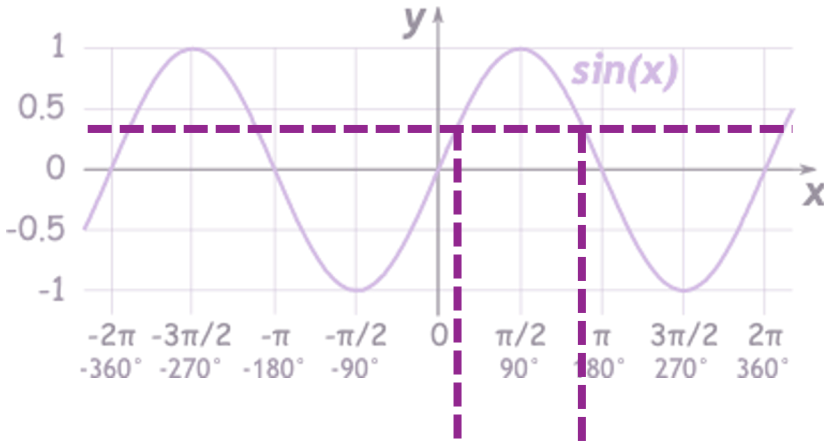






Supplementary Angles!





$$\sin(x) = \sin(y)$$

$$0° < x < y < 180°$$

Interweaving **out**

Sum-Products

Factorise $x^2 + 7x + 10$.

Find a rectangle with area 10 and perimeter 7.

Inequalities

I need to paint an area of $67\ \mathrm{m}^2$.
A can of paint covers an area of $22\ \mathrm{m}^2$.
How many cans do I need?

Pascal's Triangle

A football match finished with a a score of $3:2$. How many different ways could it have reached that result?
(e.g. $0:0 \to 0:1 \to \cdots \to 3:2$)

Other Ideas!

Remainders

Today is Saturday.
What day is it in 100 days time?

What is the difference between $\frac{100}{7}$ and $\frac{100}{7}$ rounded to the nearest whole number?

Fibonacci

In how many ways can you climb 10 steps if you can only climb 1 or 2 steps at a time?

? ? ?

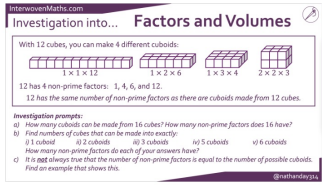
Wrapping up

interwovenmaths.com

Interwoven Maths

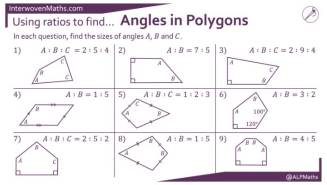
What is Interweaving? All the Tasks Submit Your Own!

Investigation into Factors and Volumes



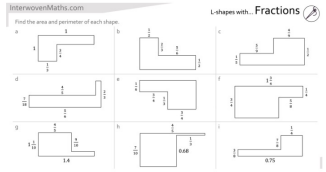
In today's Year 8 lesson I discovered a startling fact: the number of cuboids you can make from n cubes is equal to the number...

Using Ratios to Find Angles in Polygons



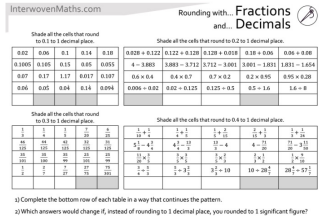
This brilliant task shared by Stephen Gregory (@ALPMaths) feels like the spiritual successor to Sam Blatherwick's wonderful task Using Ratios to Find Angles. I really...

L-shapes with Fractions

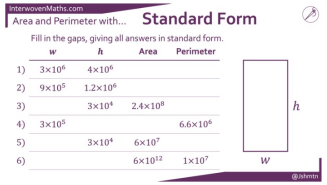


This is my second post inspired by Jo Morgan's video about using the KS2 curriculum to inform Year 7 teaching, available here. This task looks...

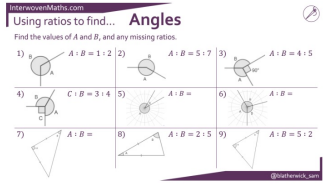
Rounding with...



Area and Perimeter with Standard Form



Using Ratios to Find Angles



This fantastic task kindly shared by

Thank you!