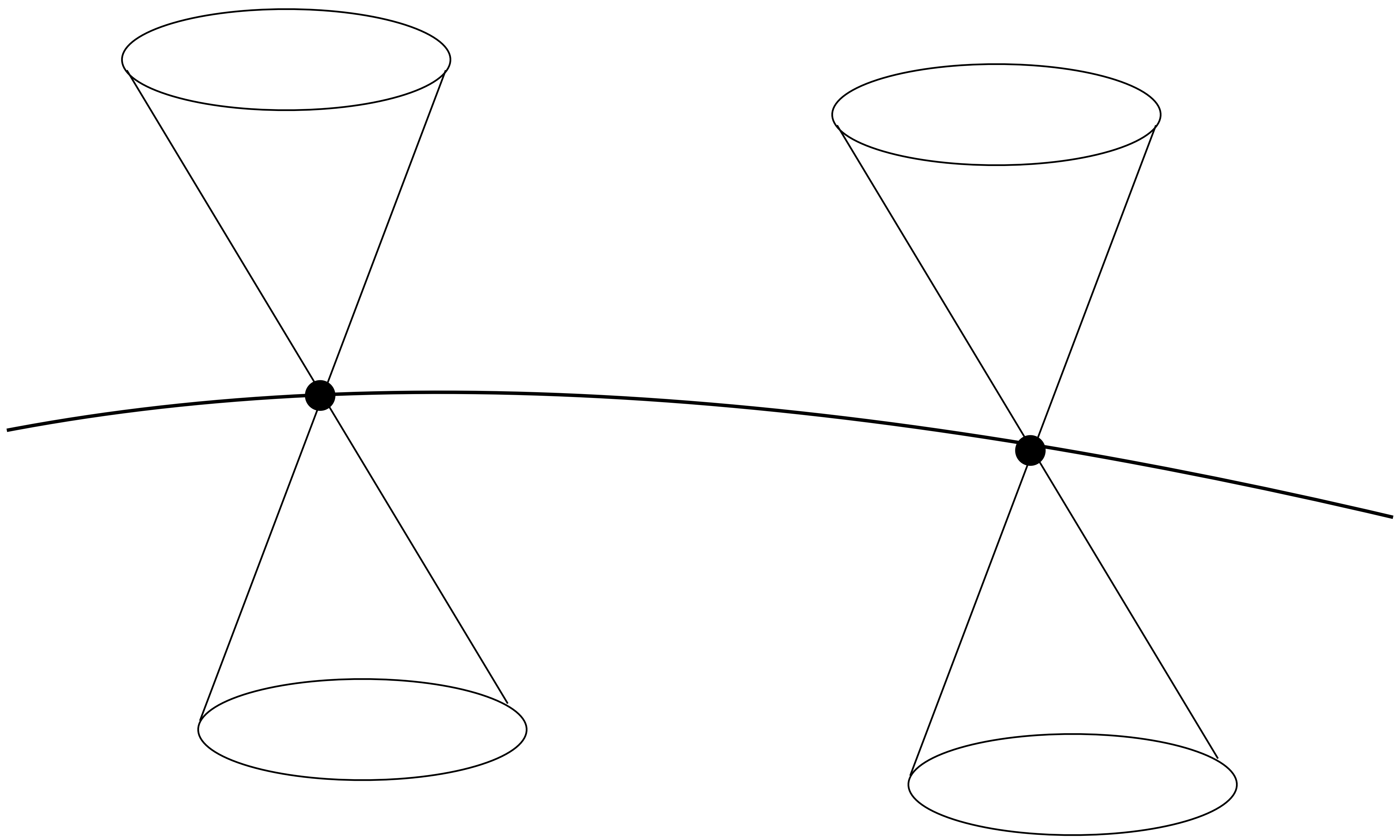
a tale of two localities

Andrea Di Biagio
A look at the interface between gravity and quantum theory
2025-07-25

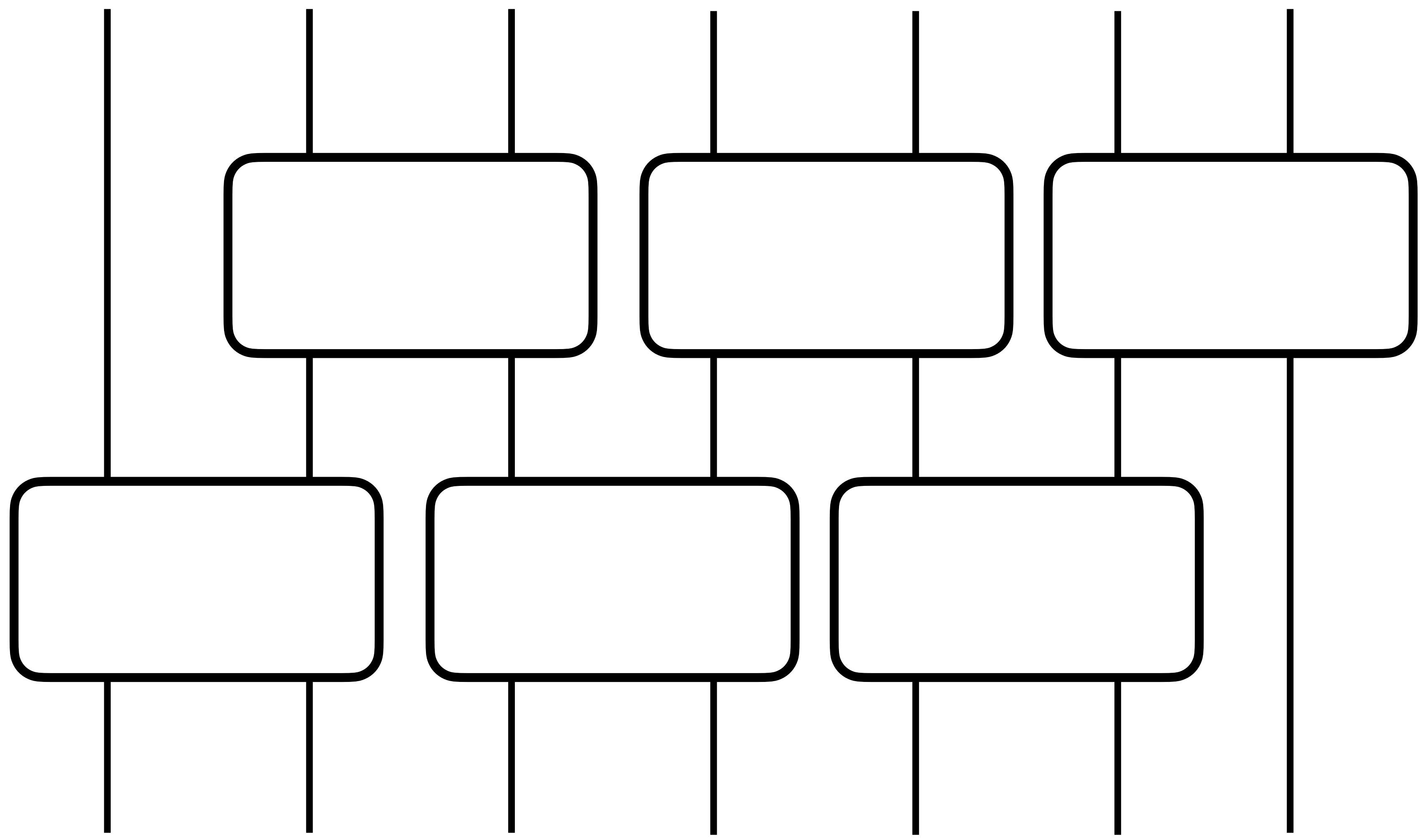
two notions of locality

relativistic



spacetime regions

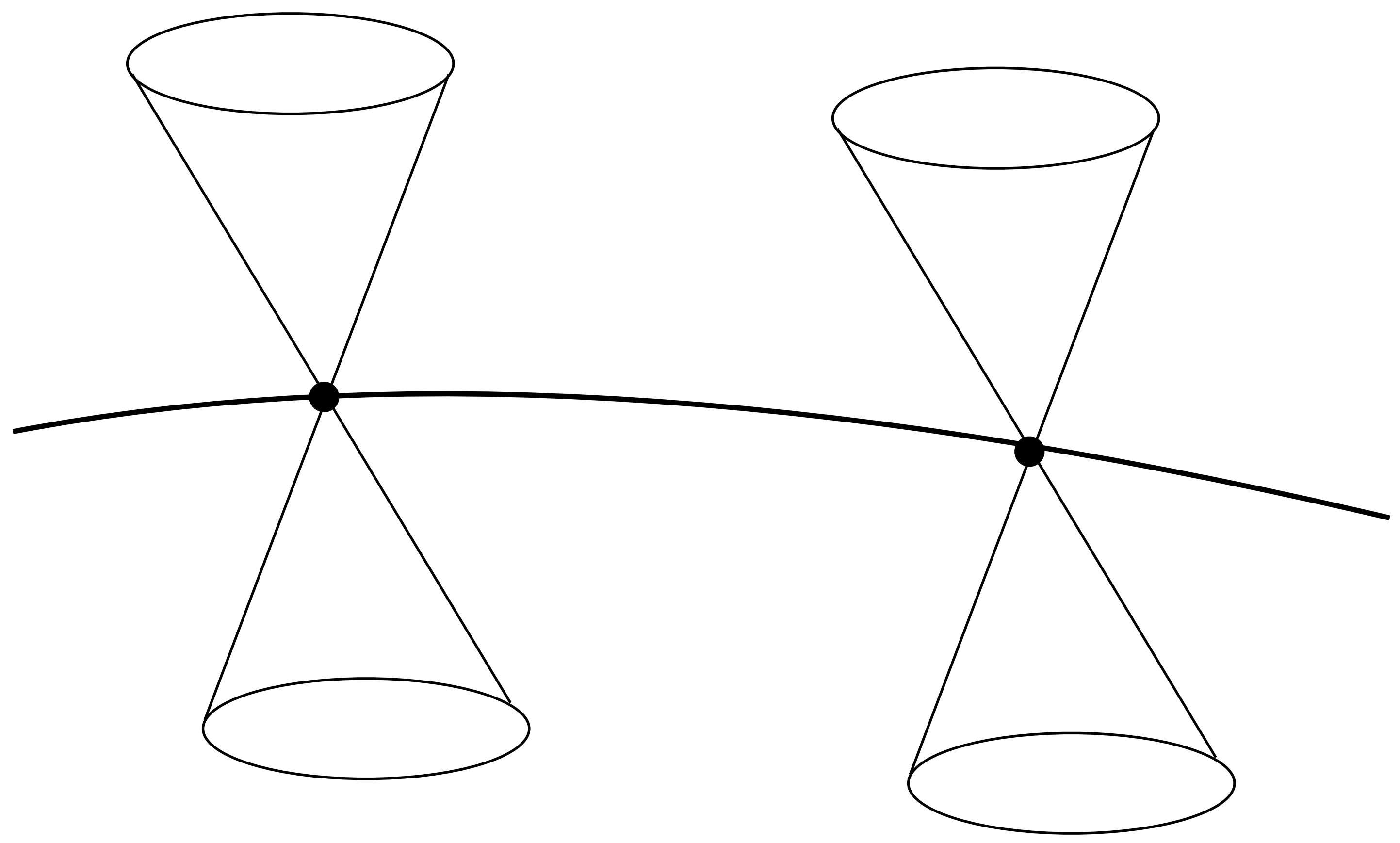
circuit



systems

two notions of locality

relativistic

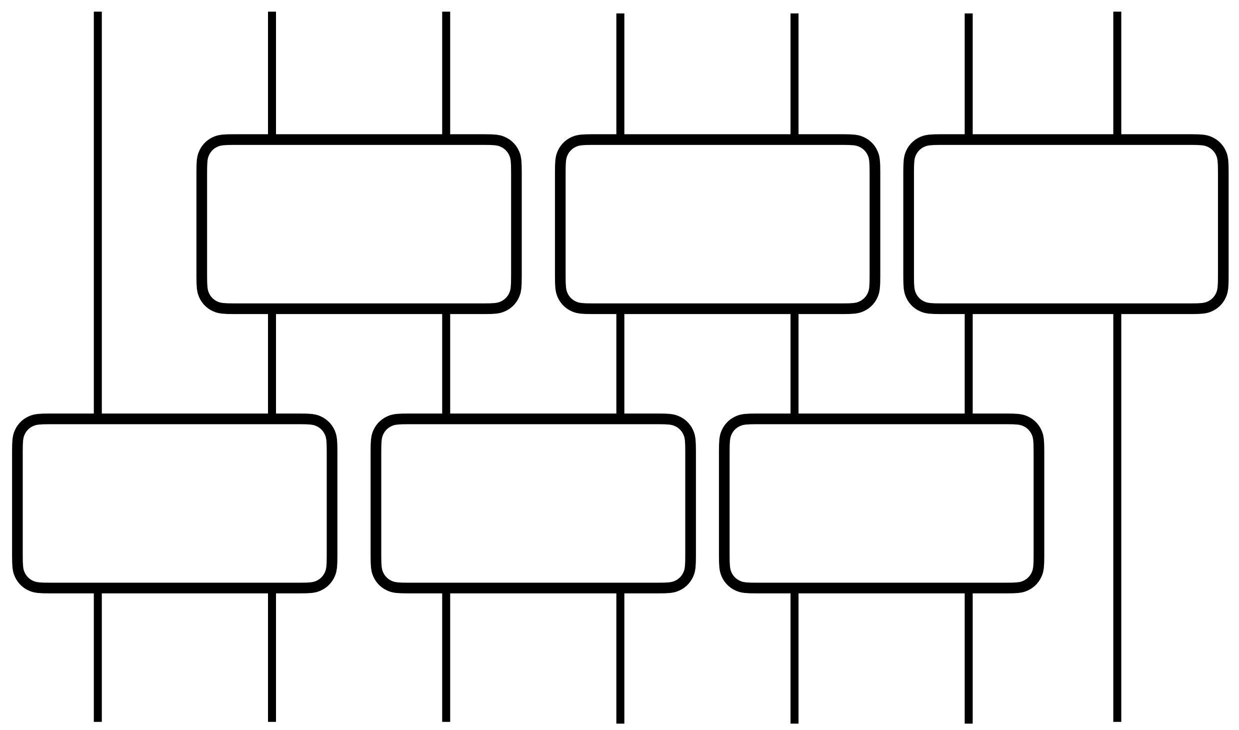


basis of relativity theory

foundational to QFT, GR

no-signalling

circuit



postulate of QM

widely used in models

assumed in reconstructions

low-energy quantum gravity

if we detect gravity mediated entanglement, then gravity cannot be both:

classical | local

Spin Entanglement Witness for Quantum Gravity

Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A. Geraci, Peter F. Barker, M. S. Kim, and Gerard Milburn

Phys. Rev. Lett. **119**, 240401 – Published 13 December 2017

QM

Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity

C. Marletto and V. Vedral

Phys. Rev. Lett. **119**, 240402 – Published 13 December 2017

constructor theory

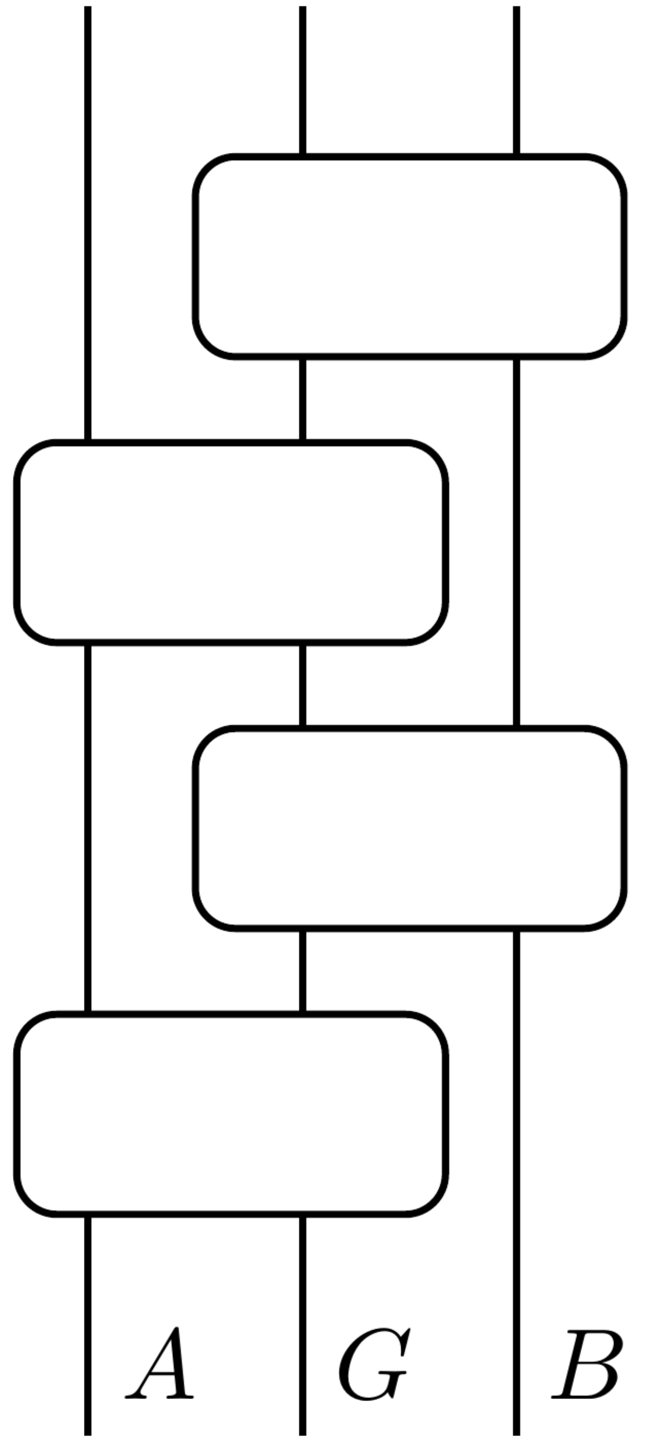
A no-go theorem on the nature of the gravitational field beyond quantum theory

Thomas D. Galley[1], Flaminia Giacomini[1], and John H. Selby[2]

Published: 2022-08-17, volume 6, page 779
Eprint: arXiv:2012.01441v7
Doi: https://doi.org/10.22331/q-2022-08-17-779
Citation: Quantum 6, 779 (2022).

GPTs

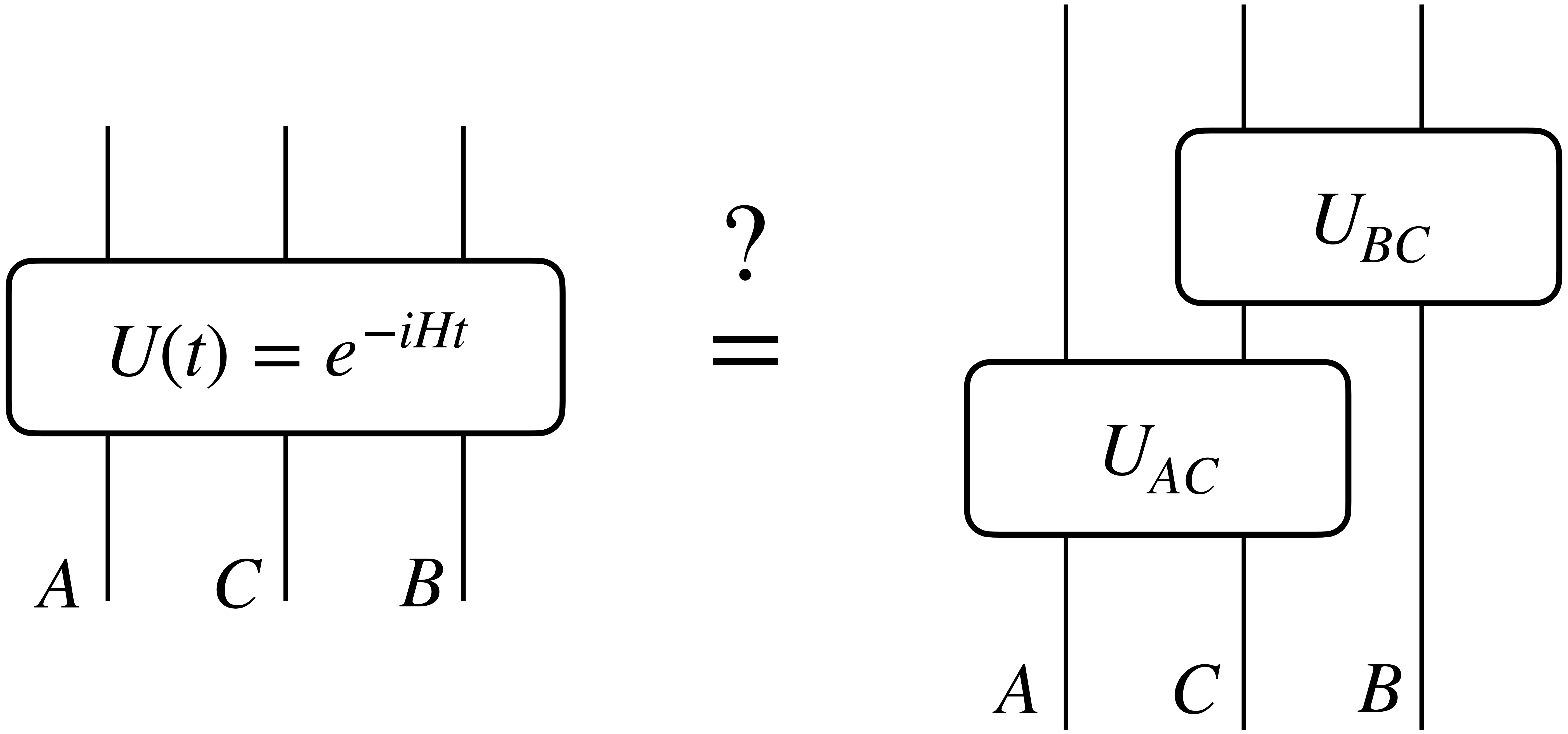
is this a good assumption?



mediation

focus on quantum theory:

$$H = H_A + H_B + H_C + H_{AC} + H_{BC}$$



plan

- **two quick observations**
- **scalar field and quantum-controlled particles**
- **only fields**
- **GME no-go theorems revisited**

} **QM+QFT**

two quick observations

observation 1

Suzuki-Trotter

$$H = H_A + H_B + H_C + H_{AC} + H_{BC}$$

observation 1

Suzuki-Trotter

$$H = H_{AC} + H_{BC}$$

$$\implies U(t) = e^{-i(H_{AC}+H_{BC})t} \quad = \quad e^{-iH_{AC}t}\, e^{-iH_{BC}t}\, e^{\frac{1}{2}[H_{AC},H_{BC}]t^2}$$

observation 1

Suzuki-Trotter

$$H = H_{AC} + H_{BC}$$

$$\implies U(t) = e^{-i(H_{AC}+H_{BC})t} \neq e^{-iH_{AC}t}\, e^{-iH_{BC}t}$$

$$= \lim_{n\to\infty} \left(e^{-iH_{AC}\, t/n} e^{-iH_{BC}\, t/n}\right)^n$$

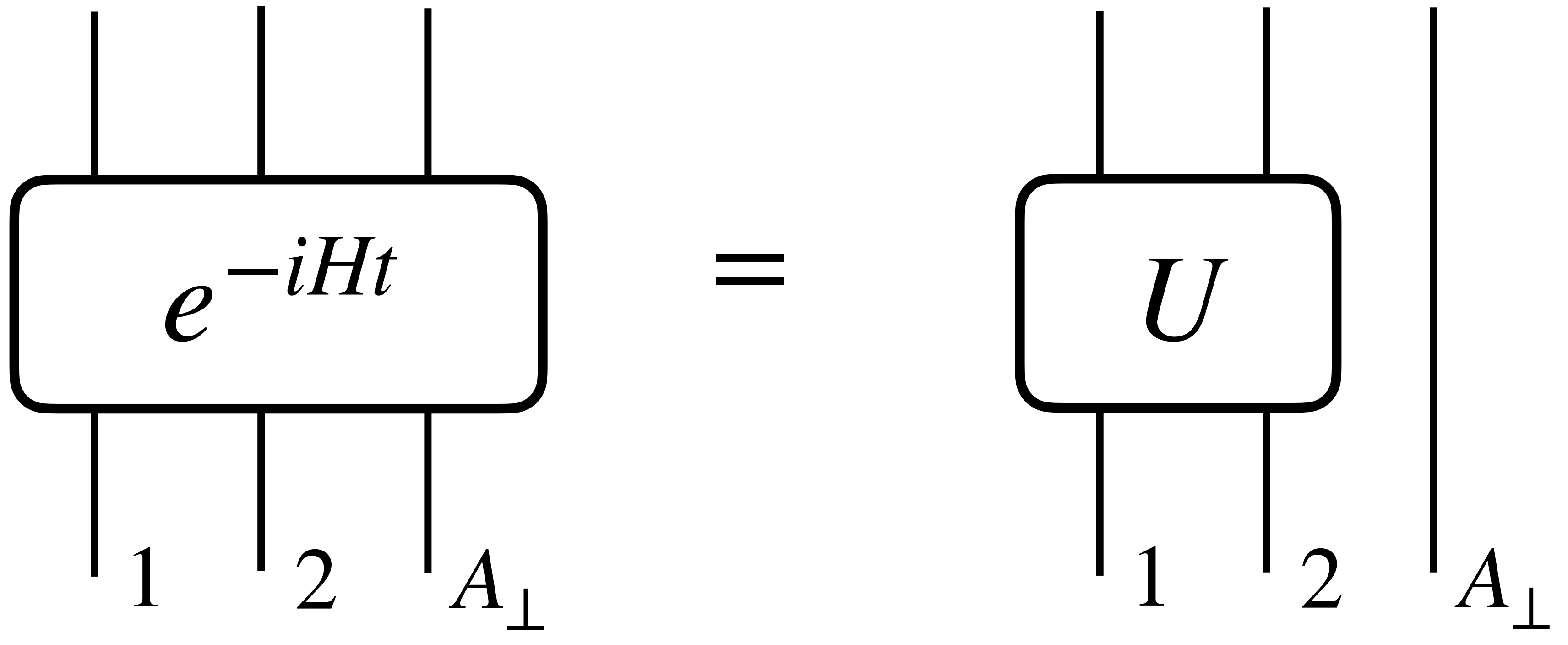
arbitrarily good approximation **but no input from relativity.**

observation 2

QED in Coulomb gauge

$$H = H_1 + H_2 + H^{\mathrm{rad}}_{A_\perp} + \frac{q_1 q_2}{|\hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2|} - \int \mathrm{d}^3\mathbf{x}\, A_\perp(\mathbf{x}) \cdot \big(J_1(\mathbf{x}) + J_2(\mathbf{x})\big)$$

$$H|\psi_1\rangle \approx \left(H_1 + H_2 + \frac{q_1 q_2}{|\hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2|}\right)|\psi_1\rangle \quad \Longrightarrow$$



$$|\psi_1\rangle \approx \frac{1}{2}\big(|L\rangle_1 + |R\rangle_1\big)\big(|L\rangle_2 + |R\rangle_2\big)|0\rangle_{A_\perp}$$

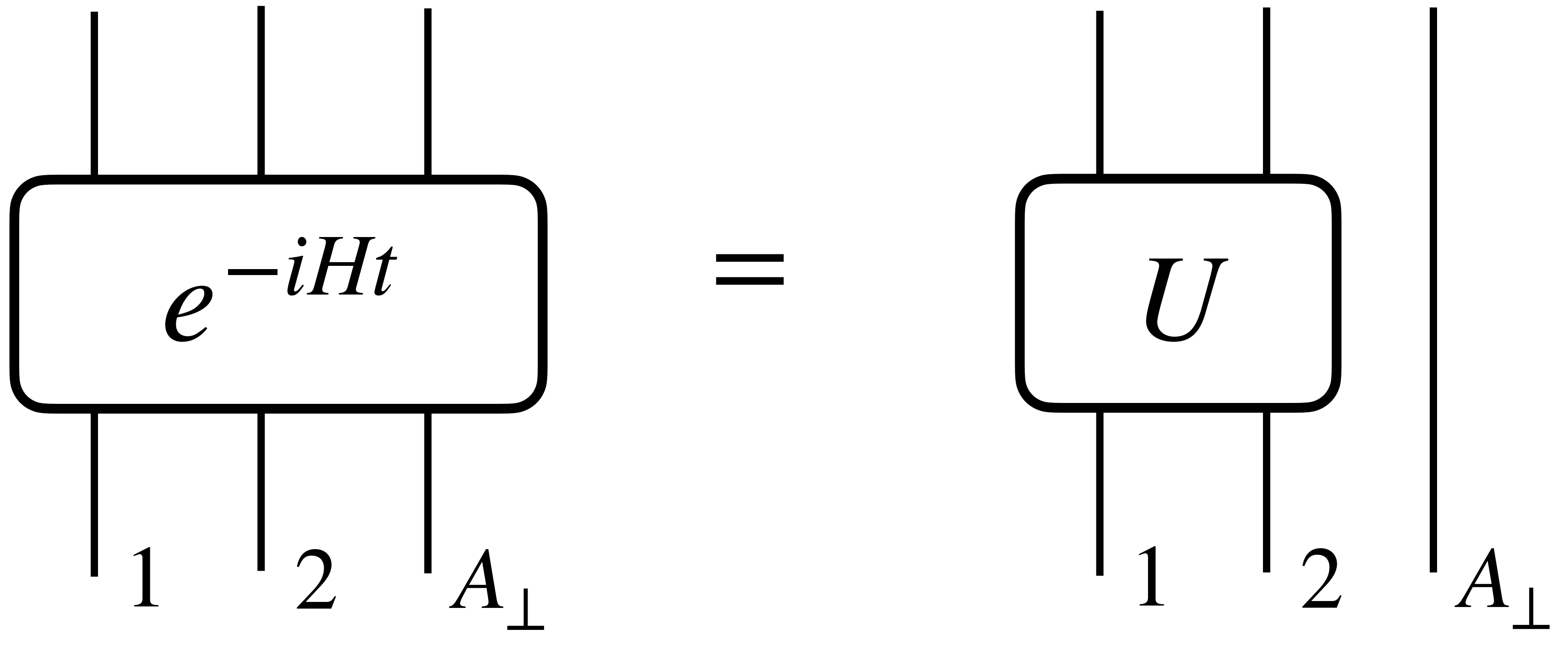
$$|\psi_0\rangle = |C\rangle_1 |C\rangle_2 |0\rangle_{A_\perp}$$

observation 2

QED in Coulomb gauge

$$H = H_1 + H_2 + H^{\mathrm{rad}}_{A_\perp} + \frac{q_1 q_2}{|\hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2|} - \int \mathrm{d}^3\mathbf{x}\, A_\perp(\mathbf{x}) \cdot \big(J_1(\mathbf{x}) + J_2(\mathbf{x})\big)$$

$$H|\psi_1\rangle \approx \left(H_1 + H_2 + \frac{q_1 q_2}{|\hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2|}\right)|\psi_1\rangle \quad \Longrightarrow$$



no mediation!

theory is relativistically local (no-signalling)

$\Longrightarrow$ mediation does not follow from relativistic locality

($\Longrightarrow$circuit locality is gauge-dependent)

On inference of quantization from gravitationally induced entanglement

Special Collection: Celebrating Sir Roger Penrose's Nobel Prize

Vasileios Fragkos; Michael Kopp; Igor Pikovski

AVS Quantum Sci. 4, 045601 (2022)

circuit locality
with massive scalar field

massive scalar field

a positive result

arXiv:2305.05645 (quant-ph)

[Submitted on 9 May 2023 (v1), last revised 14 Feb 2025 (this version, v2)]

Circuit locality from relativistic locality in scalar field mediated entanglement

Andrea Di Biagio, Richard Howl, Časlav Brukner, Carlo Rovelli, Marios Christodoulou

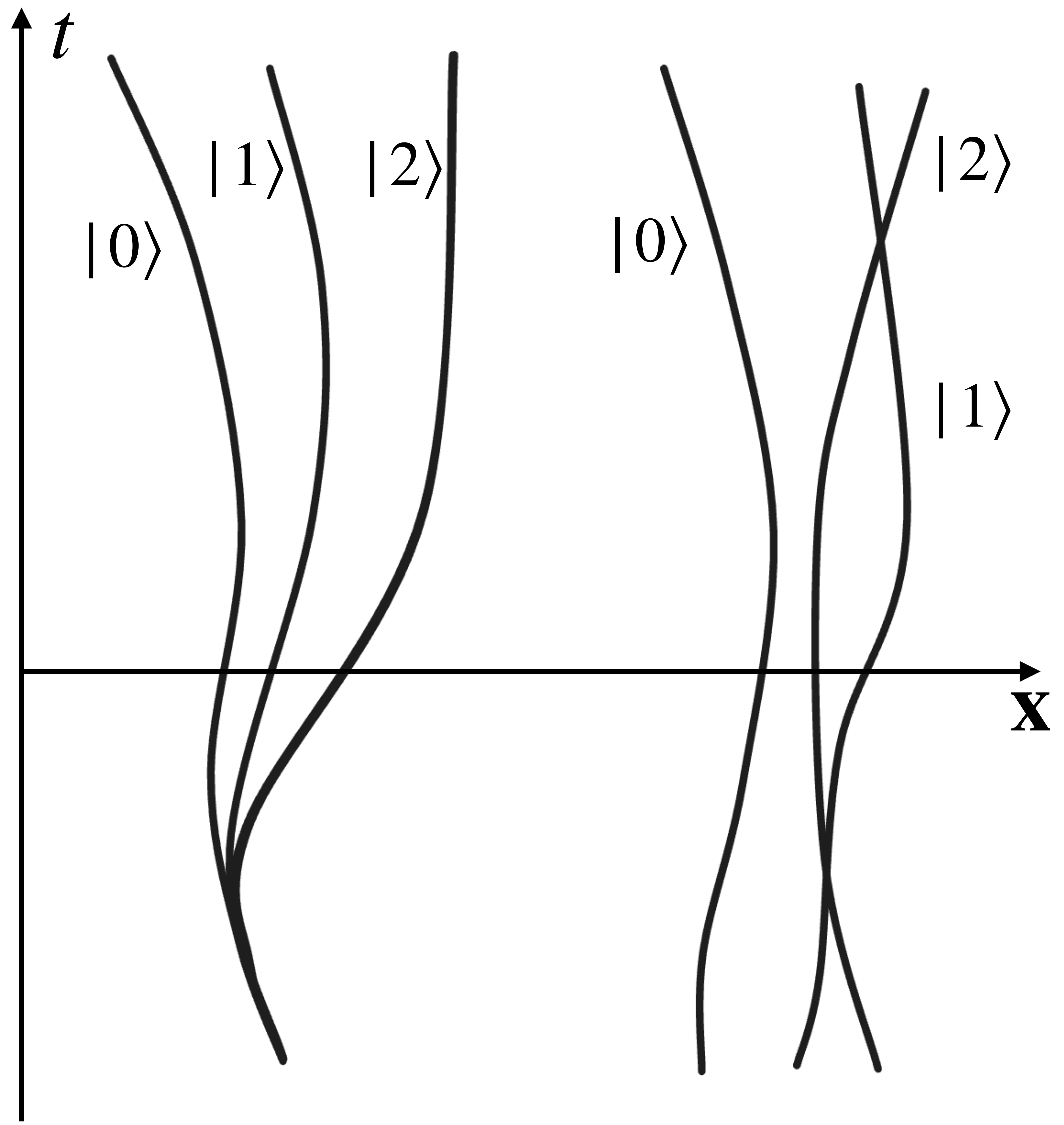
concrete example:

- **two particles coupled to a massive scalar field, in a specific regime**
- **scalar field mediates, *up to some phases***
- **microcausality ($[\hat{\phi}(x), \hat{\phi}(x')] = 0$ if x, x' spacelike) eliminates the phases**

  - **relativistic locality yields circuit locality in this approximation** massive scalar field

three key assumptions

- support of the matter wavefunctions contained within two distinct spacetime regions
- no back-action on the field
- matter in quantum-controlled superposition of semiclassical states



derivation

setup

$$\mathscr{H} = \left(\mathbb{C}^d \otimes L^2(\mathbb{R}^3)\right) \otimes \left(\mathbb{C}^d \otimes L^2(\mathbb{R}^3)\right) \otimes \mathscr{F}_\phi$$

$$\hat{H}(t) = \hat{H}_A(t) + \hat{H}_B(t) + \hat{H}_\phi + \hat{H}_{\text{int}}$$

$$\hat{H}_A(t) = \sum_r |r\rangle\langle r| \otimes \hat{H}^r_A(t)$$

quantum-controlled dynamics

$$\hat{H}_\phi = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \omega_{\mathbf{k}} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}}$$

kinetic field term

$$\hat{H}_{\text{int}} = \int \mathrm{d}^3\mathbf{x}\, \hat{\phi}(\mathbf{x}) \left(\hat{\mu}_A(\mathbf{x}) + \hat{\mu}_B(\mathbf{x})\right)$$

$$\hat{\mu}_A(\mathbf{x}) = \mu(\mathbf{x} - \hat{\mathbf{x}}_A)$$

local interaction

derivation

qudit-controlled dynamics

no back action on the qudits + matter in superposition of pointer states:

$$|\Psi(t)\rangle = \sum_{rs} c_{rs} |rs\rangle |\psi_A^r(t)\rangle |\psi_B^s(t)\rangle |\phi^{rs}(t)\rangle$$

particles: $$\frac{\mathrm{d}}{\mathrm{d}t} |\psi_A^r(t)\rangle = -i\hat{H}_A^r(t) |\psi_A^r(t)\rangle$$

field: $$\frac{\mathrm{d}}{\mathrm{d}t} |\phi^{rs}(t)\rangle = -i\left(\hat{H}_\phi + \hat{H}_{\mathrm{int}}^{rs}(t)\right) |\phi^{rs}(t)\rangle$$

$$\hat{H}_{\mathrm{int}}^{rs}(t) = \langle \psi_A^r(t)\psi_B^s(t) | \hat{H}_{\mathrm{int}} | \psi_A^r(t)\psi_B^s(t)\rangle$$

evolution of the whole system: $$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}_A^r \otimes \hat{U}_B^s \otimes \hat{U}_\phi^{rs}$$

derivation

condition for subsystem locality

$$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}_A^r \otimes \hat{U}_B^s \otimes \hat{U}_\phi^{rs}$$

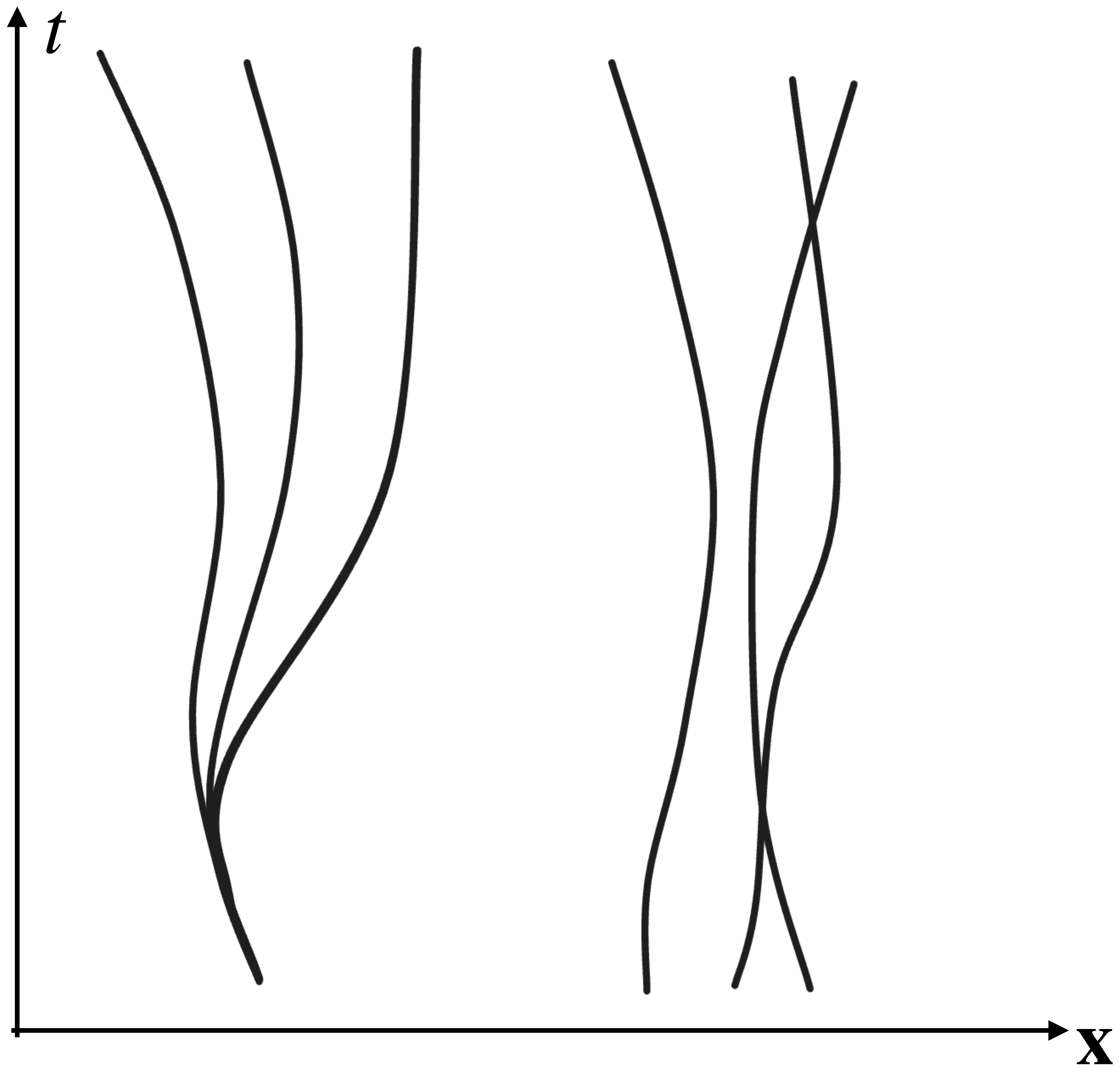
is not field a mediation yet

but if we had $\forall rs \;:\; \hat{U}_\phi^{rs} = \hat{U}_\phi^r \circ \hat{U}_\phi^s$ **then it would be:**

$$\hat{U} = \left(\sum_s |s\rangle\langle s| \otimes \hat{U}_B^s \otimes \hat{U}_\phi^s \right) \circ \left(\sum_r |r\rangle\langle r| \otimes \hat{U}_A^r \otimes \hat{U}_\phi^r \right)$$

derivation

evolution of the field

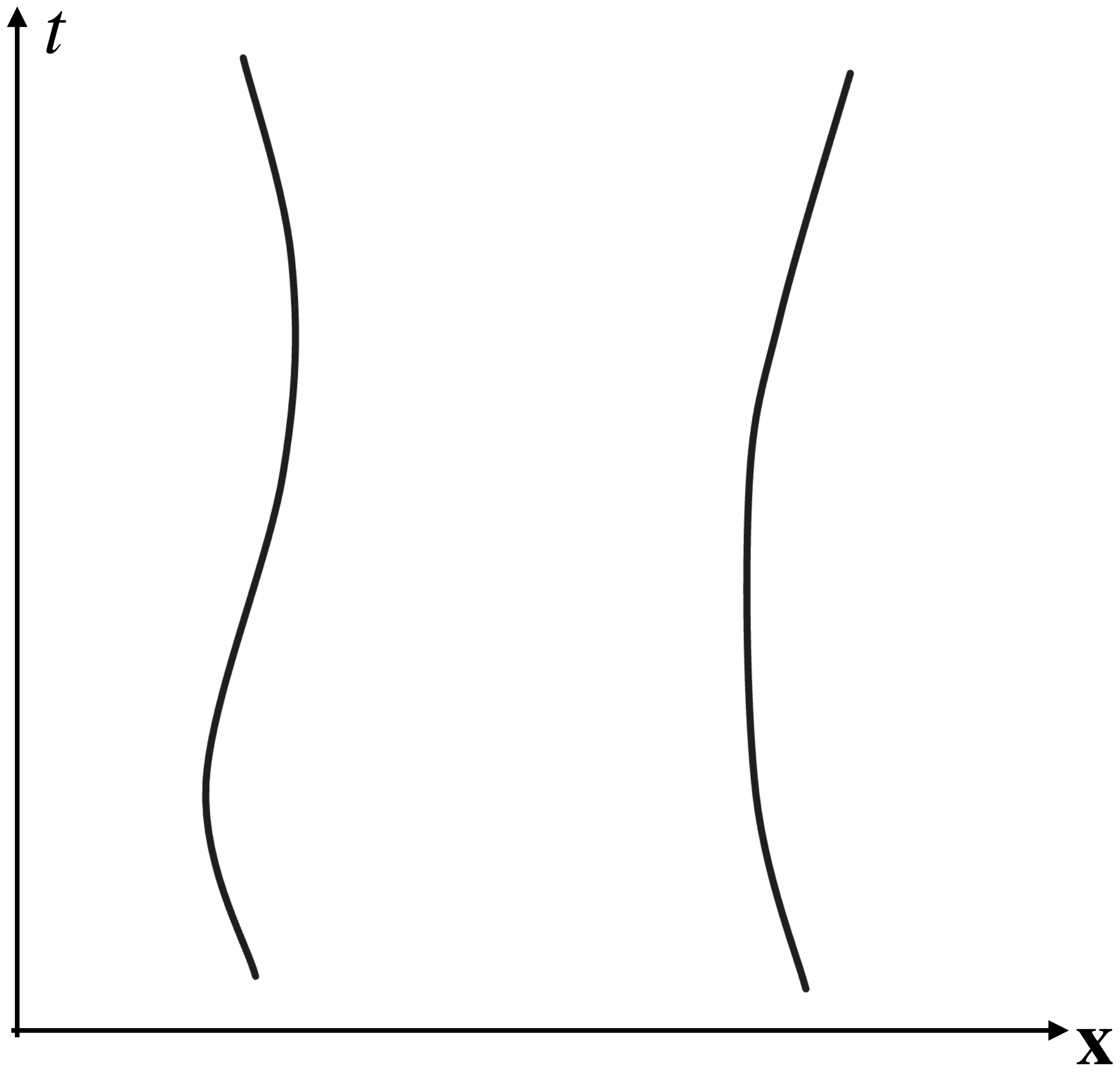


$$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}^r_A \otimes \hat{U}^s_B \otimes \hat{U}^{rs}_\phi$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}^{rs}_\phi(t) = -i\hat{H}^{rs}(t)\hat{U}^{rs}_\phi(t)$$

derivation

evolution of the field



quantum field with classical source!

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}^{rs}_{\phi}(t) = -i\hat{H}^{rs}(t)\hat{U}^{rs}_{\phi}(t)$$

$$\hat{H}^{rs}(t) = \hat{H}_{\phi} + \langle \psi^r_A(t)\psi^s_B(t) \,|\, \hat{H}_{\mathrm{int}} \,|\, \psi^r_A(t)\psi^s_B(t) \rangle$$

exact solution

$$\hat{U}^{rs}_{\phi} = e^{i\Omega^{rs}}\hat{D}^{rs}e^{-i\hat{H}_{\phi}(t_2 - t_1)}$$

derivation

field mediation?

$$\hat{U}^{rs}_{\phi} = e^{i\Omega^{rs}}\hat{D}^{rs}e^{-i\hat{H}_0(t_2-t_1)} = e^{i\tilde{\Omega}^{rs}}\hat{U}^{r}_{\phi}\hat{U}^{s}_{\phi}e^{-i\hat{H}_0(t_2-t_1)}$$

full evolution:

$$\hat{U} = \sum_{sr} \boxed{e^{i\tilde{\Omega}^{rs}}} \left(|s\rangle\langle s| \otimes \hat{U}^{s}_{B} \otimes \hat{U}^{s}_{\phi} \right) \circ \left(|r\rangle\langle r| \otimes \hat{U}^{r}_{A} \otimes \hat{U}^{r}_{\phi} \right) \circ e^{-i\hat{H}_0(t_2-t_1)}$$

almost there!

derivation

the phase

$$\mu_A^r(t,\mathbf{x}) = \langle \psi_A^r(t) \,|\, \hat{\mu}_A(\mathbf{x}) \,|\, \psi_A^r(t) \rangle$$

$$\tilde{\Omega}^{rs} = -\, i \iint_{t_1}^{t_2} \mathrm{d}t\mathrm{d}t' \iint \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{x}'\, \mu_A^r(t,\mathbf{x})\mu_B^s(t',\mathbf{x}')[\hat{\phi}_I(t,\mathbf{x}), \hat{\phi}_I(t',\mathbf{x}')]$$

$$-i \int_{t_1}^{t_2} \mathrm{d}t \int_{t_1}^{t} \mathrm{d}t' \iint \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{x}' \Big(\mu_A^r(t,\mathbf{x})\mu_B^s(t',\mathbf{x}') + \mu_B^r(t,\mathbf{x})\mu_A^s(t',\mathbf{x}') \Big) [\hat{\phi}_I(t,\mathbf{x}), \hat{\phi}_I(t',\mathbf{x}')]$$

derivation

relativistic locality

$$\mu_A^r(t, \mathbf{x}) = \langle \psi_A^r(t) \,|\, \hat{\mu}_A(\mathbf{x}) \,|\, \psi_A^r(t) \rangle$$

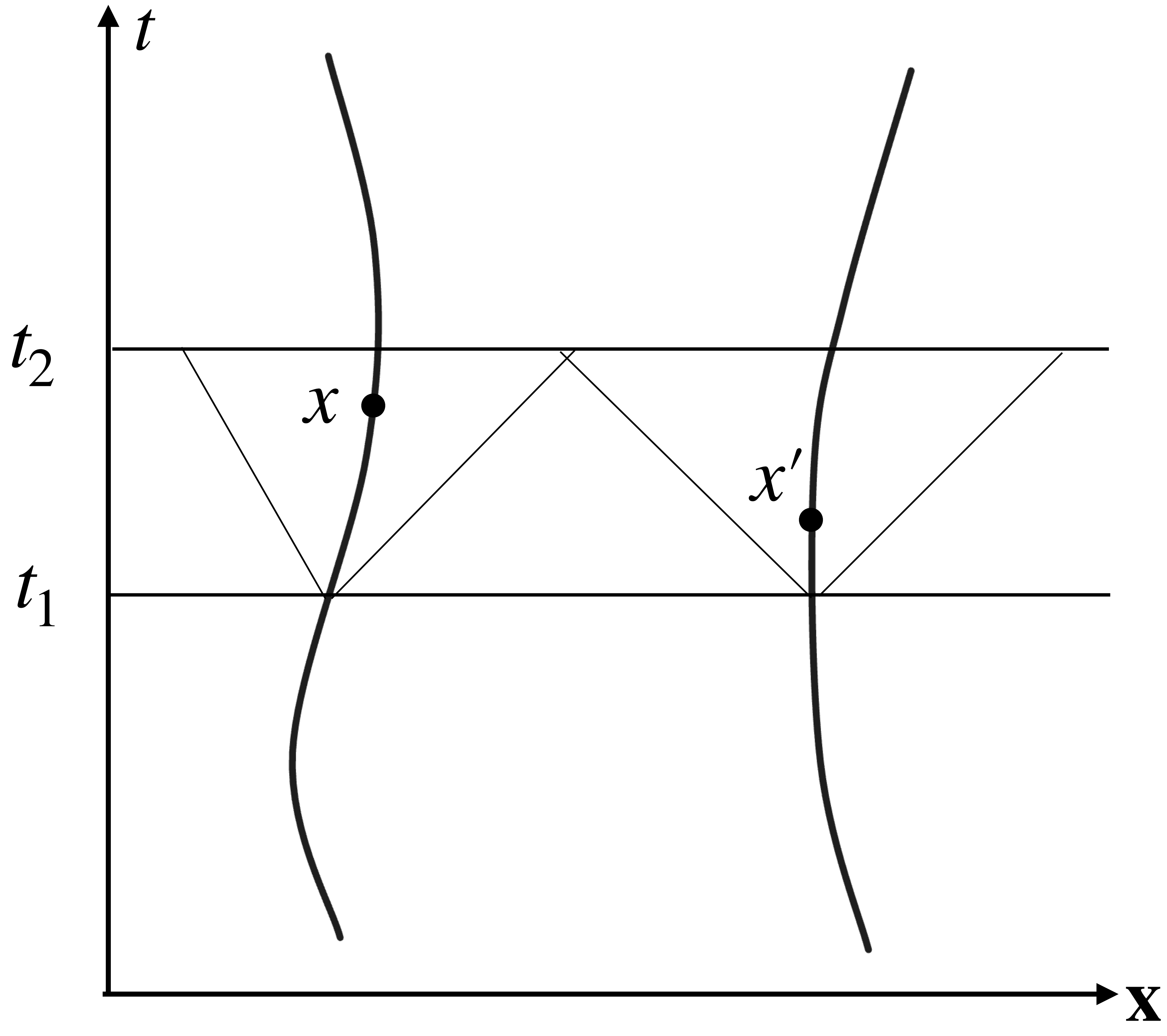
$$\tilde{\Omega}^{rs} = -i \iint_{t_1}^{t_2} \mathrm{d}^4x \mathrm{d}^4x \, \mu_A^r(x) \mu_B^s(x') [\hat{\phi}_I(x), \hat{\phi}_I(x')] - \cdots$$

microcausality:

$[\hat{\phi}_I(x), \hat{\phi}_I(x')] = 0$ **if** x **and** x' **are spacelike**

if $\operatorname{supp} \mu_A^r, \quad \operatorname{supp} \mu_B^s$ **are spacelike**

then $\tilde{\Omega}^{rs} = 0$



derivation

relativistic locality

$$\mu^r_A(t, \mathbf{x}) = \langle \psi^r_A(t) \,|\, \hat{\mu}_A(\mathbf{x}) \,|\, \psi^r_A(t) \rangle$$

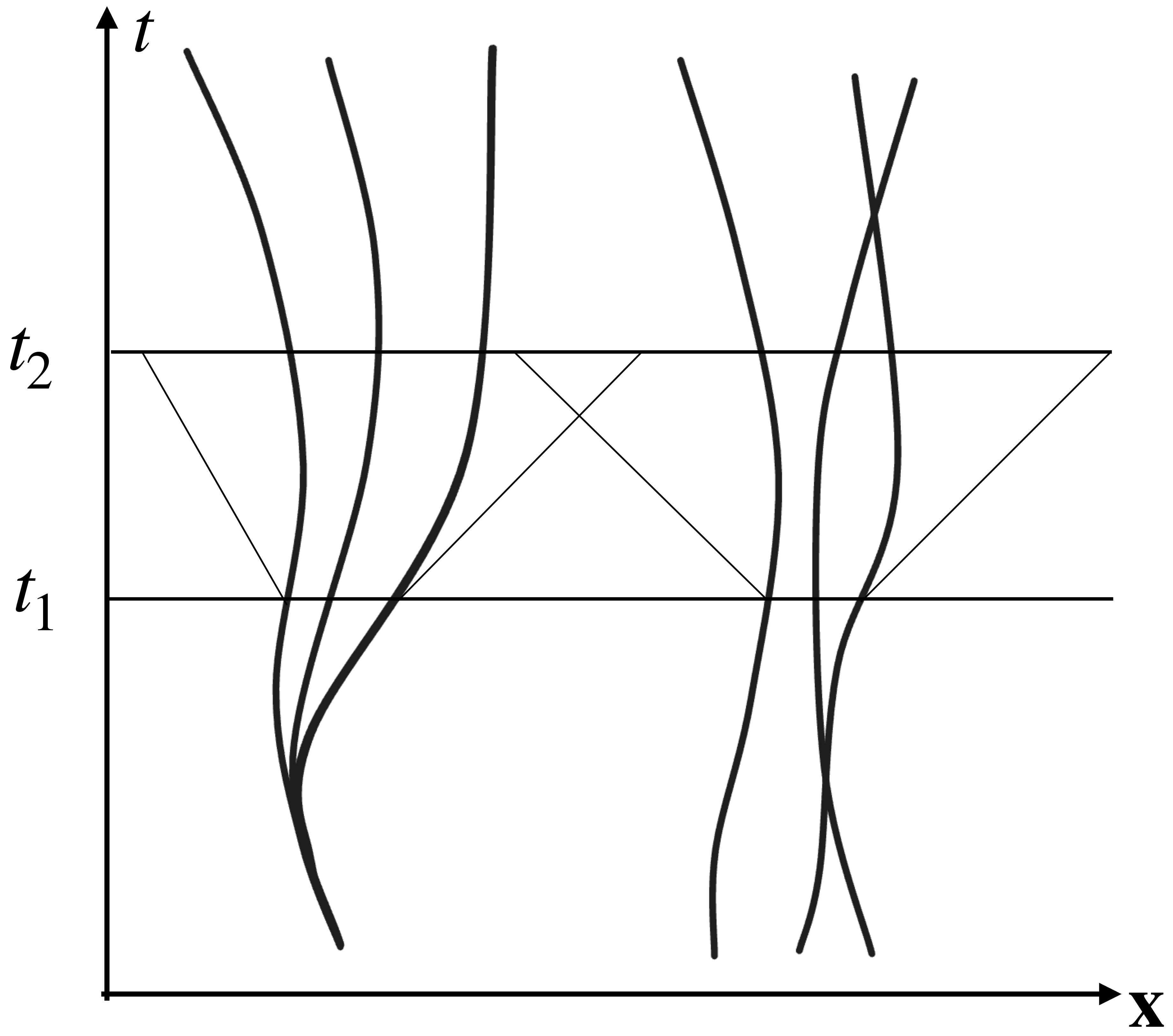
$$\tilde{\Omega}^{rs} = -\, i \iint_{t_1}^{t_2} \mathrm{d}^4x \mathrm{d}^4x \, \mu^r_A(x) \mu^s_B(x') [\hat{\phi}_I(x), \hat{\phi}_I(x')] - \cdots$$

microcausality:

$[\hat{\phi}_I(x), \hat{\phi}_I(x')] = 0$ **if x and x' are spacelike**

if $\mathrm{supp}\, \mu^r_A$, $\mathrm{supp}\, \mu^s_B$ **are spacelike** $\forall r, s$

then $\tilde{\Omega}^{rs} = 0 \qquad \forall r, s$

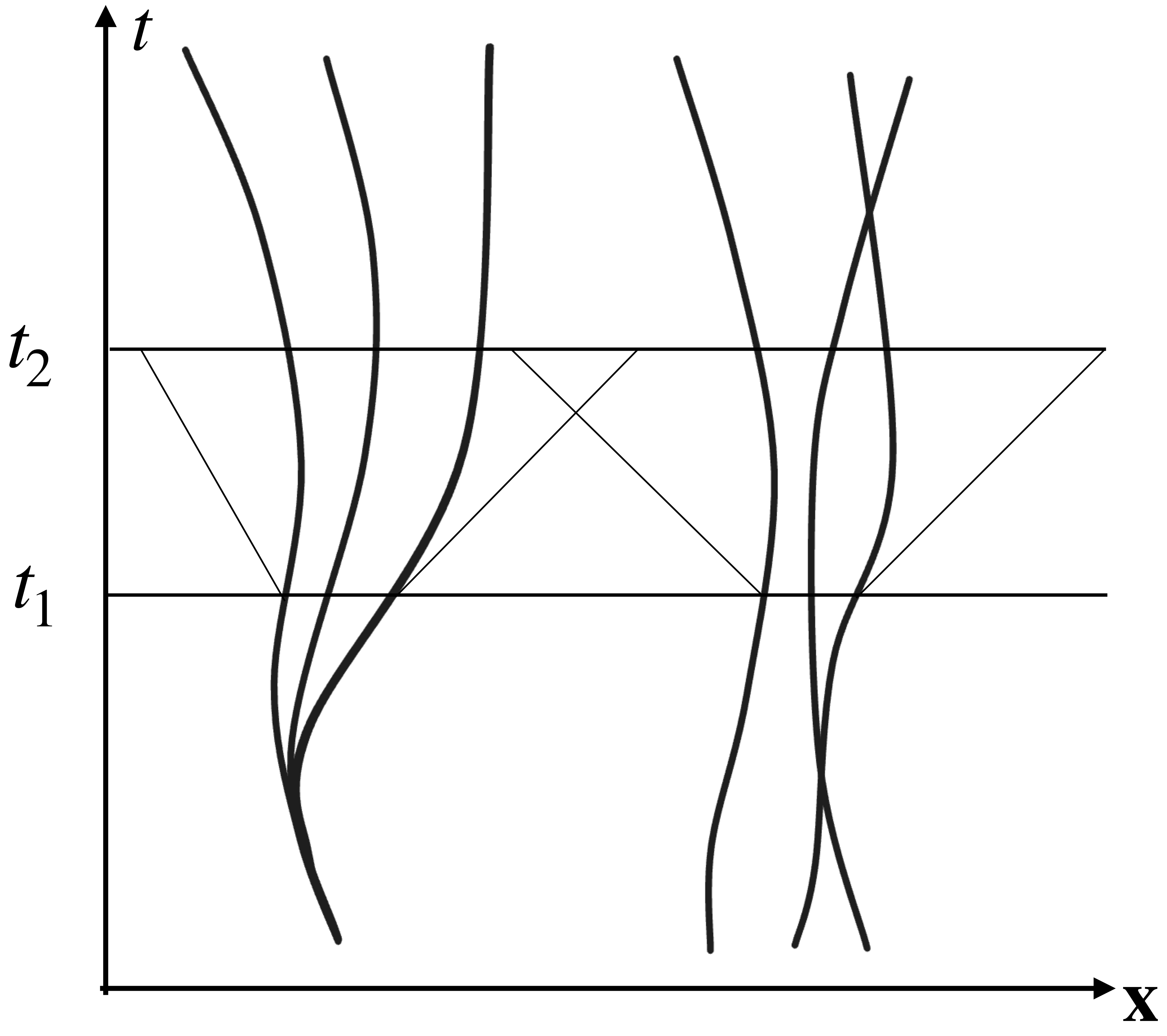


derivation

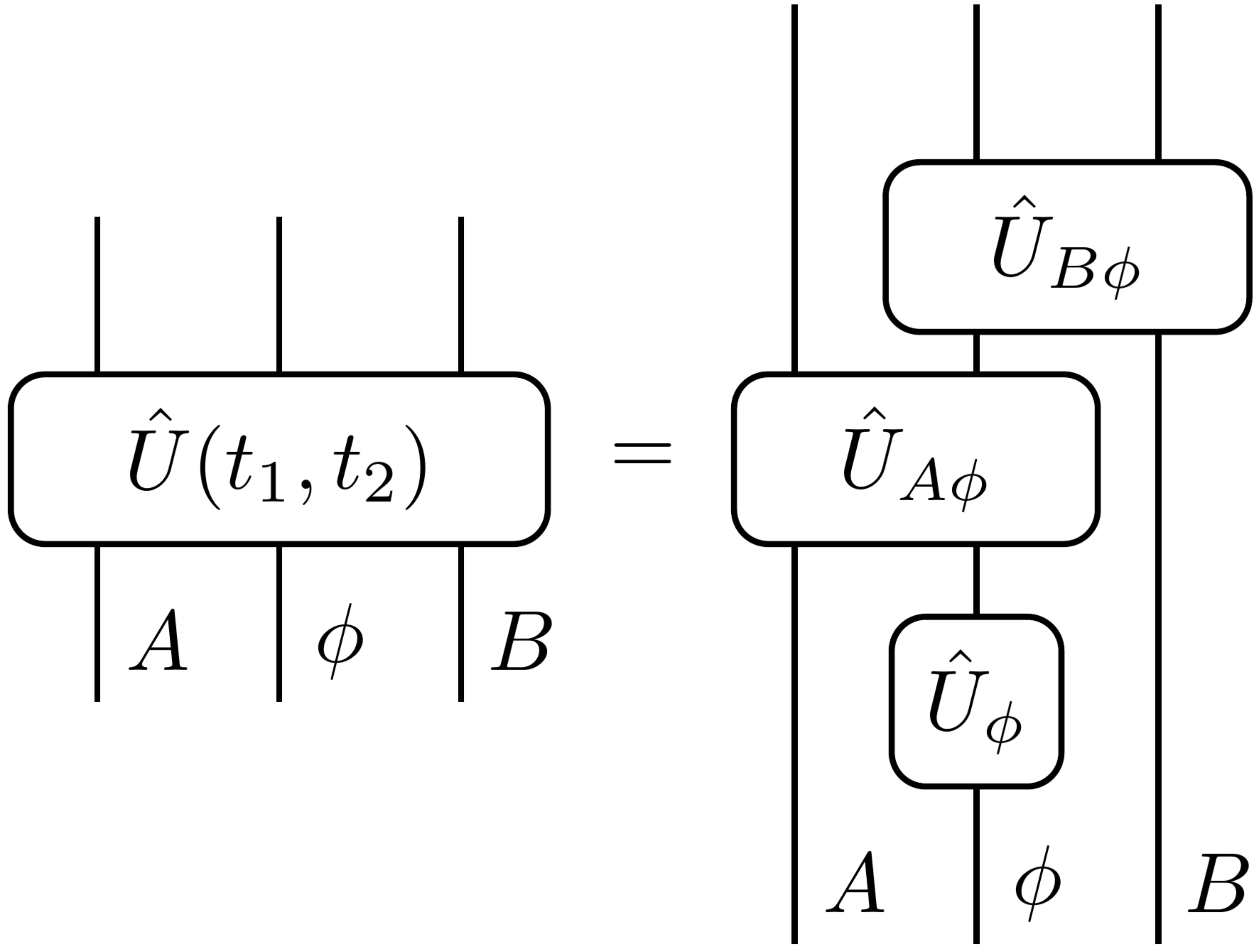
relativistic locality gives circuit locality

if $\mathrm{supp}\,\mu_A^r, \quad \mathrm{supp}\,\mu_B^s$ **are spacelike** $\forall r, s$

then $\hat{U}(t_1, t_2) = \hat{U}_{B\phi} \circ \hat{U}_{A\phi} \circ e^{-i\hat{H}_0(t_2 - t_1)}$

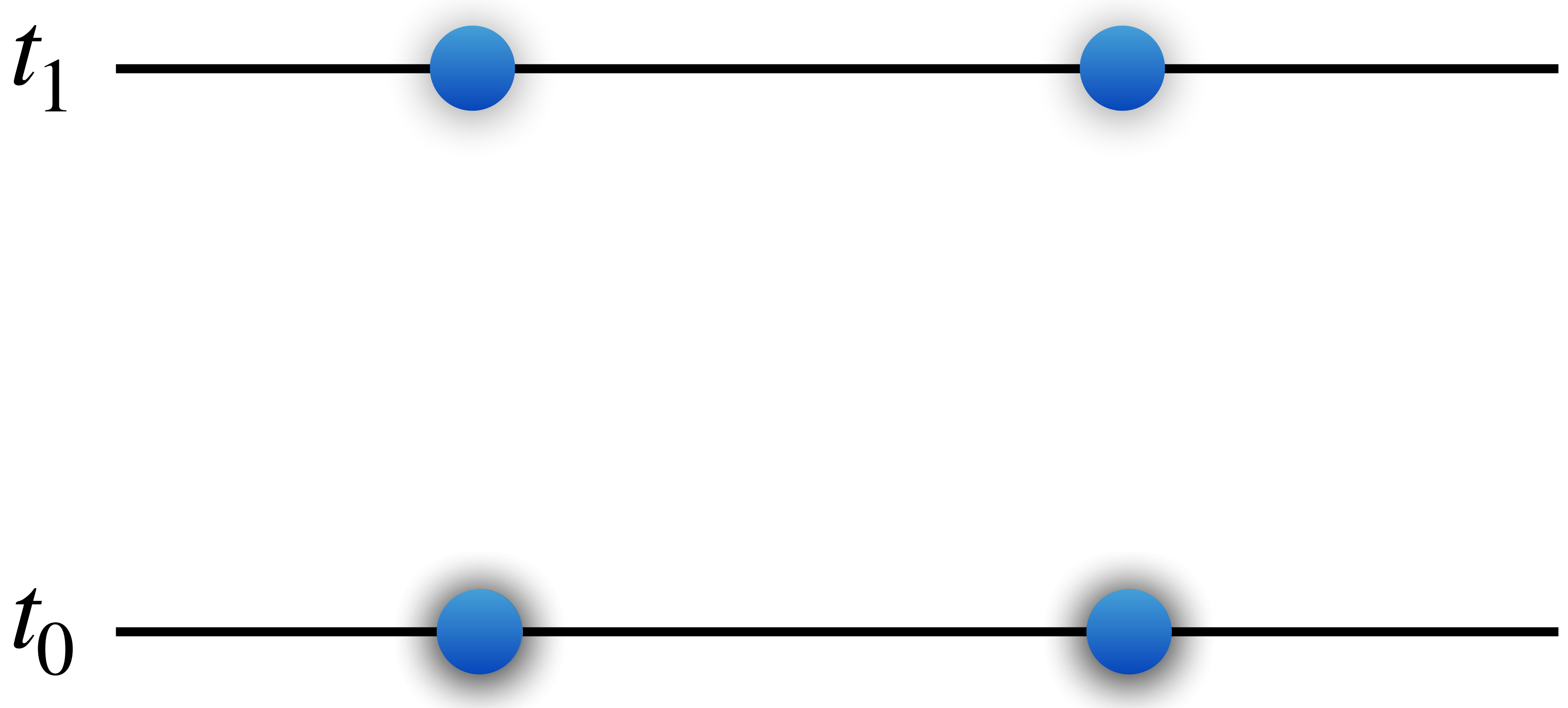


$\Longrightarrow$



derivation

scalar-field mediated entanglement



$$|\Psi_f\rangle = \frac{1}{2}\sum_{rs=0,1} e^{i\theta_{rs}}|rs\rangle|\psi_{A,f}\rangle|\psi_{B,f}\rangle|\phi_f^{rs}\rangle$$

coherent state peaked around classical solution

$$|\Psi_0\rangle = \frac{1}{2}\sum_{rs=0,1} |rs\rangle|\psi_{A,0}\rangle|\psi_{B,0}\rangle|\phi_0\rangle$$

$$\langle\phi_f^{rs}|\phi^{r's'}\rangle \approx 1 \implies |\Psi_f\rangle \approx \left(\frac{1}{2}\sum_{rs=0,1} e^{i\theta^{rs}}|rs\rangle\right)\otimes|\psi_{AB\phi}\rangle \qquad \theta^{rs} = -\frac{1}{2}\int \mathrm{d}^4x\,\rho^{rs}(x)\phi^{rs}(x) = S_\phi^{sr}$$

Locally Mediated Entanglement in Linearized Quantum Gravity

Marios Christodoulou[1,2], Andrea Di Biagio[1,3], Markus Aspelmeyer[1,2,4], Časlav Brukner[1,2,4], Carlo Rovelli[5,6,7], and Richard Howl[8,9]

Phys. Rev. Lett. 130, 100202 – **Published 10 March, 2023**

entangling phases θ^{rs} are given by the on-shell action of field sourced by classical trajectory!

only fields

only fields

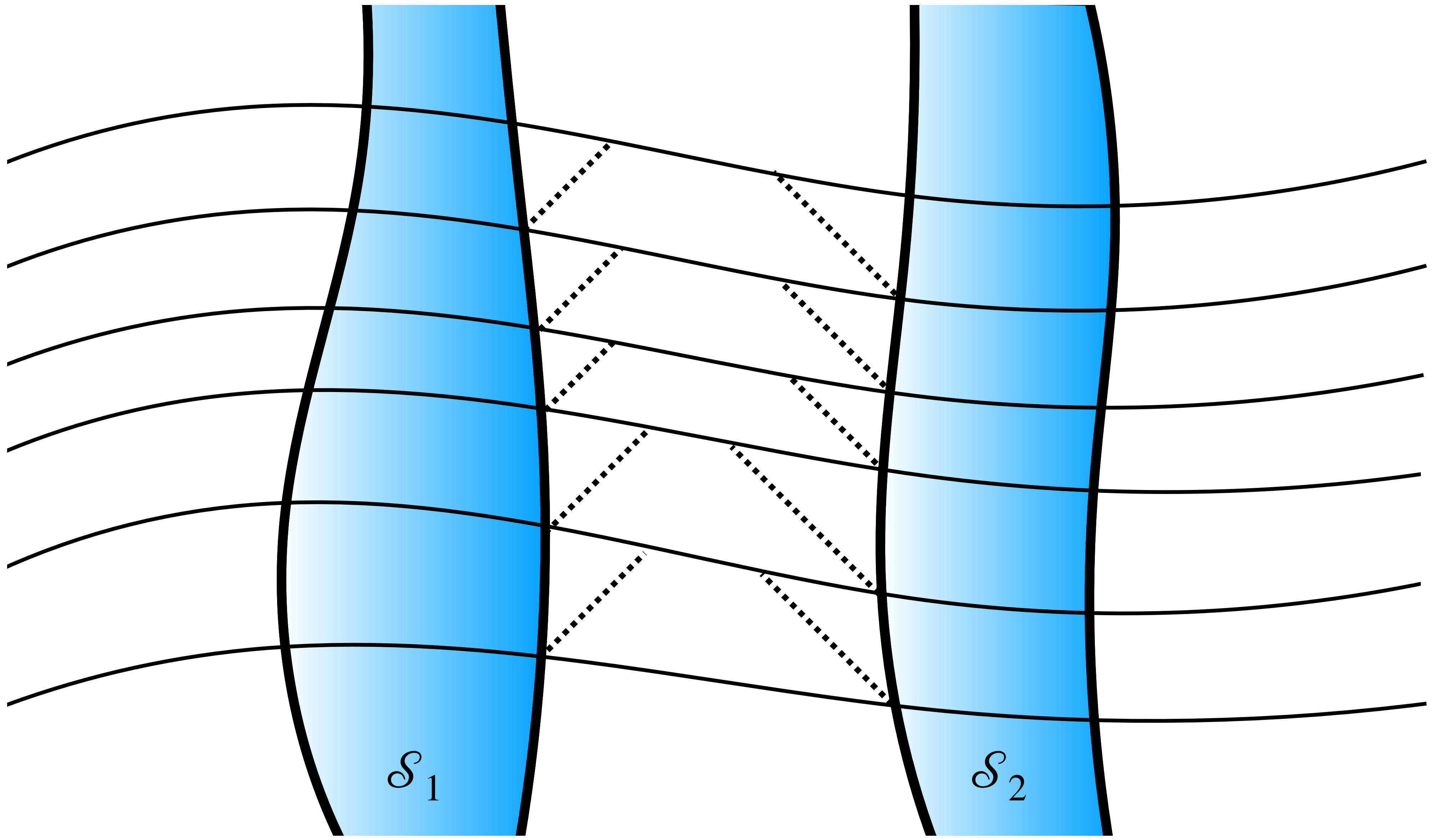
a more general result

(work in progress)

T. Rick Perche

Marios Christodoulou

- three fields interacting with each other
- assume localisation condition on the fields
- microcausality ($[\hat{\phi}(x), \hat{\phi}(x')] = 0$ if x, x' spacelike)
- valid for possibly curved spacetime



only fields

mediation in QFT

$$\mathscr{L} = \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_\varphi + \mathscr{L}_{1\varphi} + \mathscr{L}_{2\varphi}$$

$\mathscr{L} = \mathscr{L}(\mathbf{x})$

local
Lagrangian density

free-field term
for each field

$-\mathcal{O}_\varphi \cdot J_1$ $\quad -\mathcal{Q}_\varphi \cdot J_2$

local interactions

$$[\hat{A}(x), \hat{B}(y)] = 0$$

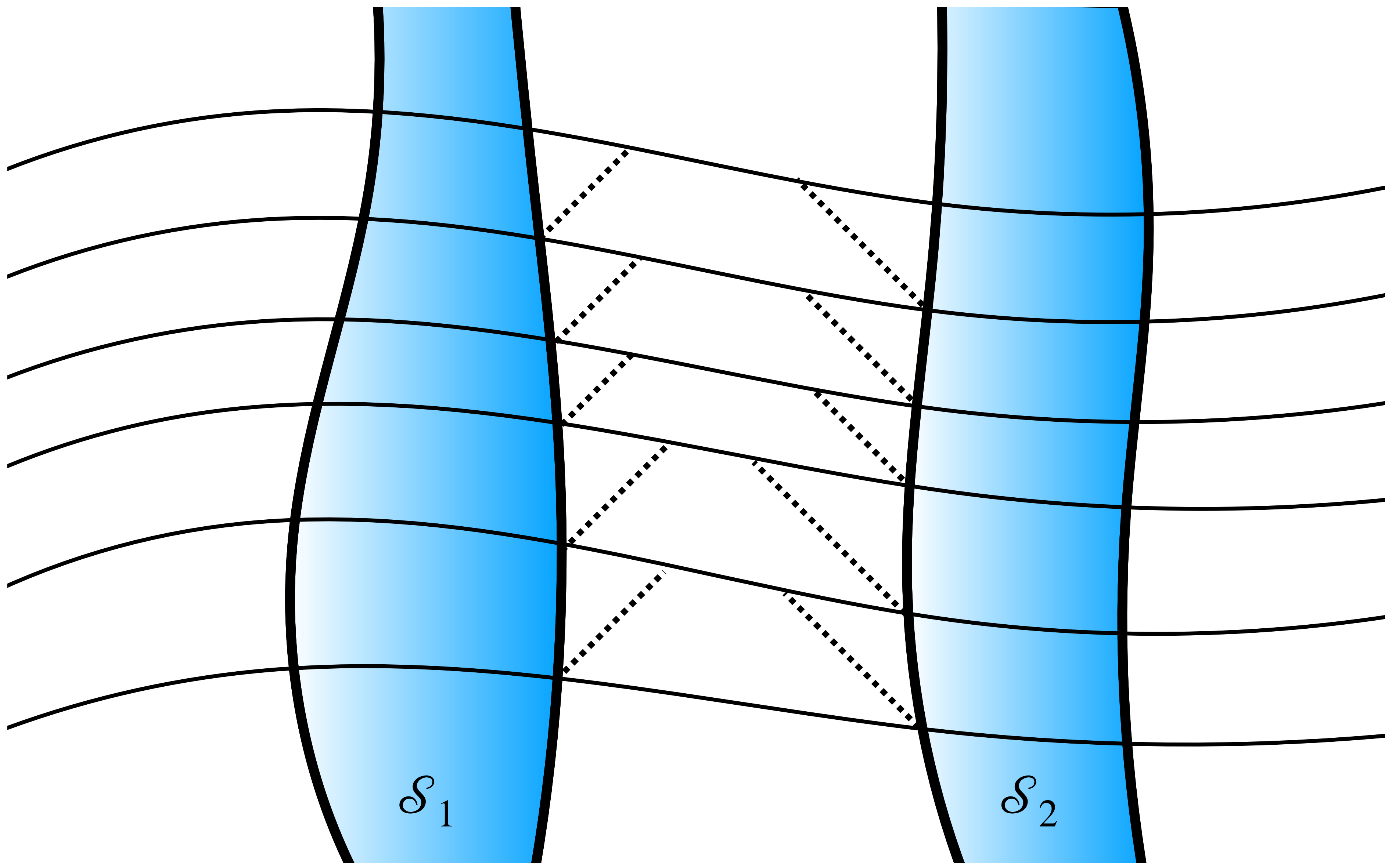
if x and y spacelike

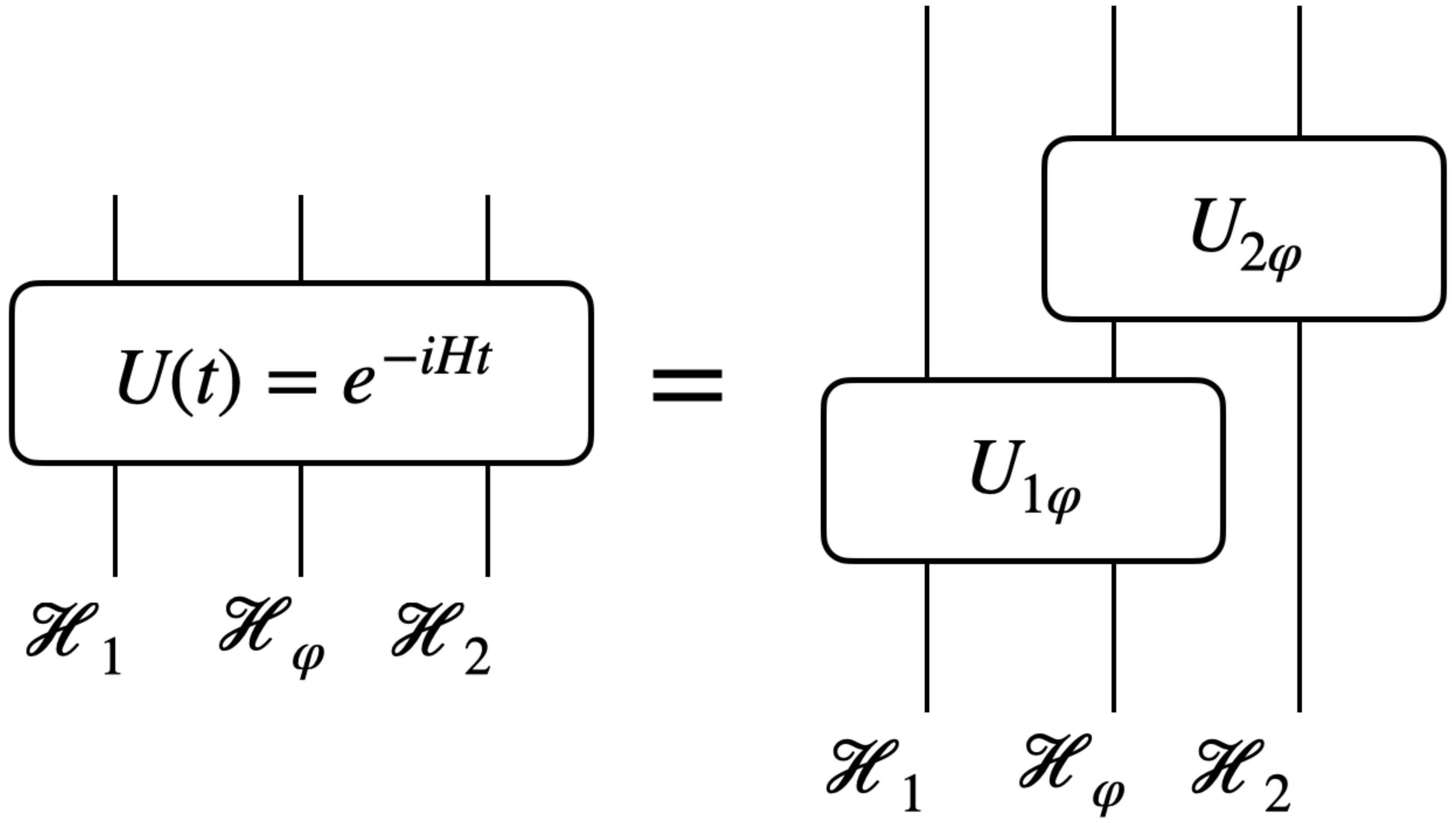
microcausality

$\hat{\mathcal{O}}_1(x) = 0 \quad$ if $x \notin \mathcal{S}_1$

$\hat{\mathcal{O}}_2(x) = 0 \quad$ if $x \notin \mathcal{S}_2$

localisation

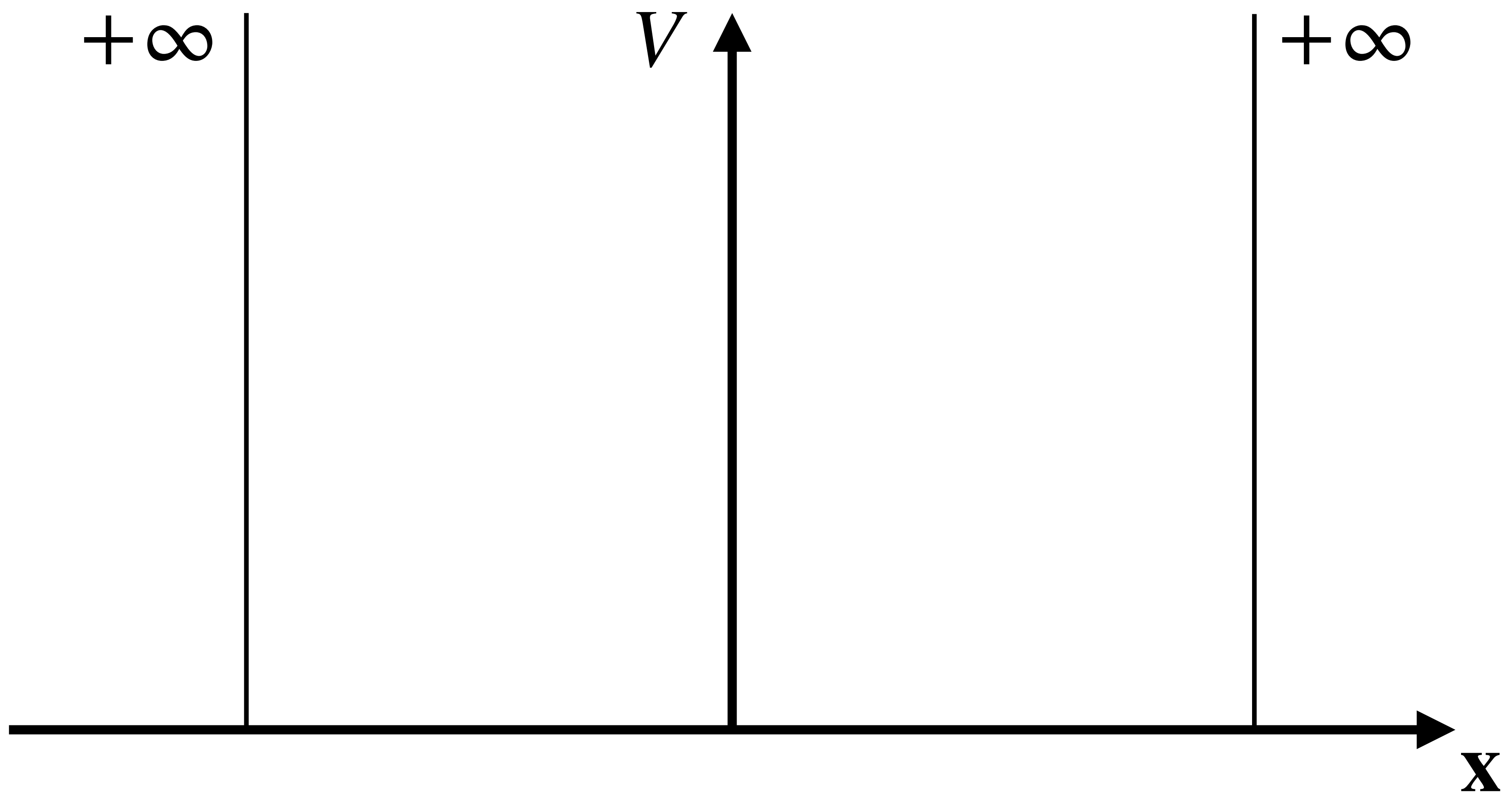




only fields

localisation assumption

$$\mathscr{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi - V(\mathbf{x})\phi$$



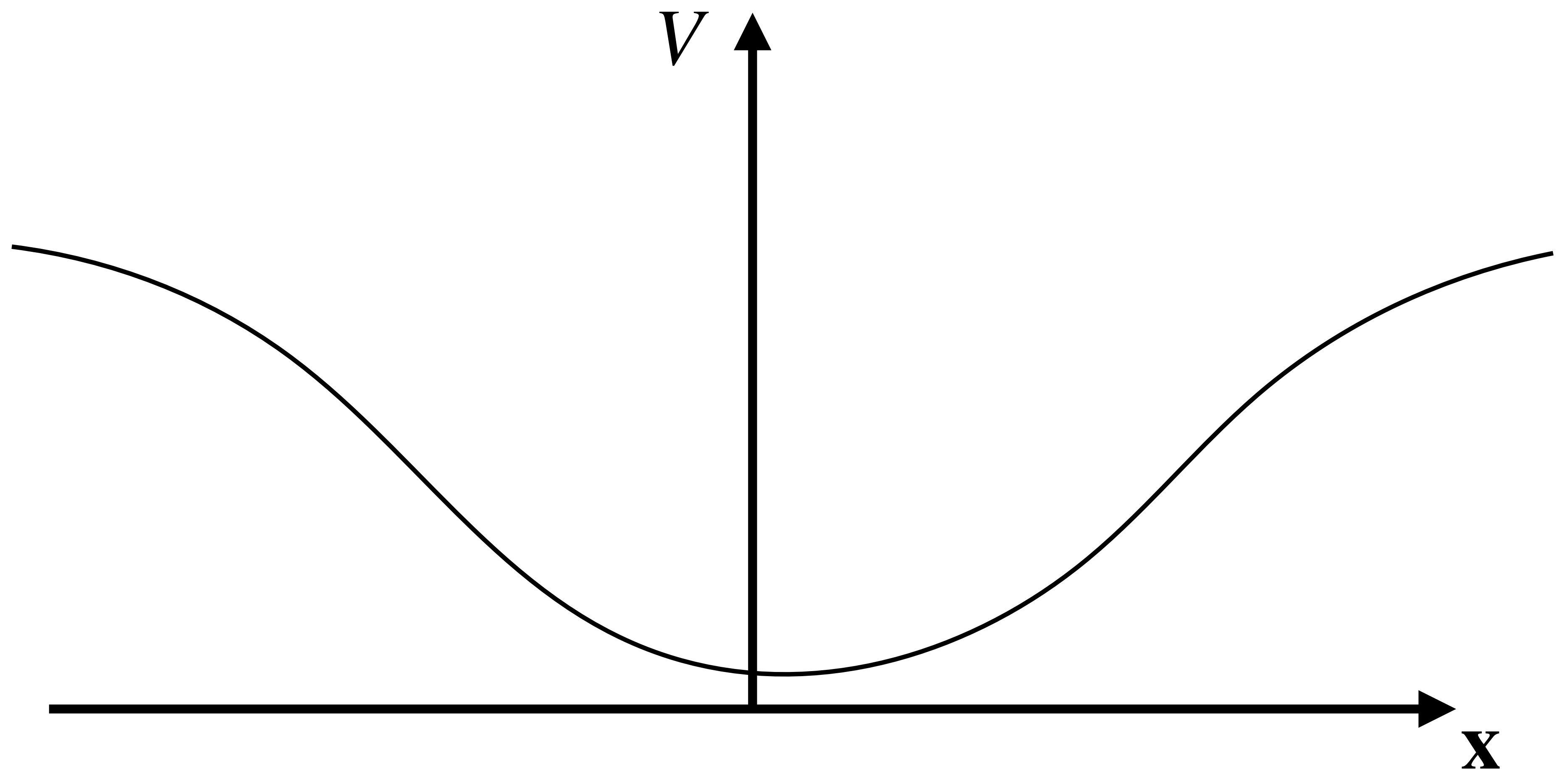
$$\hat{\phi}(x) = \sum_n \left(e^{-i\omega_n t}\phi_n(\mathbf{x})\hat{a}_n + e^{i\omega_n t}\phi_n^*(\mathbf{x})\hat{a}_n^\dagger\right)$$

$\phi_n(\mathbf{x}) = 0$ **if** $\mathbf{x} \notin \mathscr{S}$ **and then** $\hat{\phi}(\mathbf{x}) = 0$

only fields

localisation assumption

$$\mathscr{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi - V(\mathbf{x})\phi$$



$$\hat{\phi}(x) = \sum_n \left(e^{-i\omega_n t}\phi_n(\mathbf{x})\hat{a}_n + e^{i\omega_n t}\phi_n^*(\mathbf{x})\hat{a}_n^\dagger\right) + \int \mathrm{d}^3k \left(e^{-i\omega_\mathbf{k} t}\phi_\mathbf{k}(\mathbf{x})\hat{a}_\mathbf{k} + e^{i\omega_\mathbf{k} t}\phi_\mathbf{k}^*(\mathbf{x})\hat{a}_\mathbf{k}^\dagger\right)$$

$\phi_n(\mathbf{x}) \sim 0$ as $\mathbf{x} \notin \mathscr{S}$

$\phi_\mathbf{k}(\mathbf{x}) \nsim 0$ unnormalisable

if $a_\mathbf{k}|\psi\rangle = 0$ then $\hat{\phi}(\mathbf{x})|\psi\rangle \sim 0$ as $\mathbf{x} \notin \mathscr{S}$

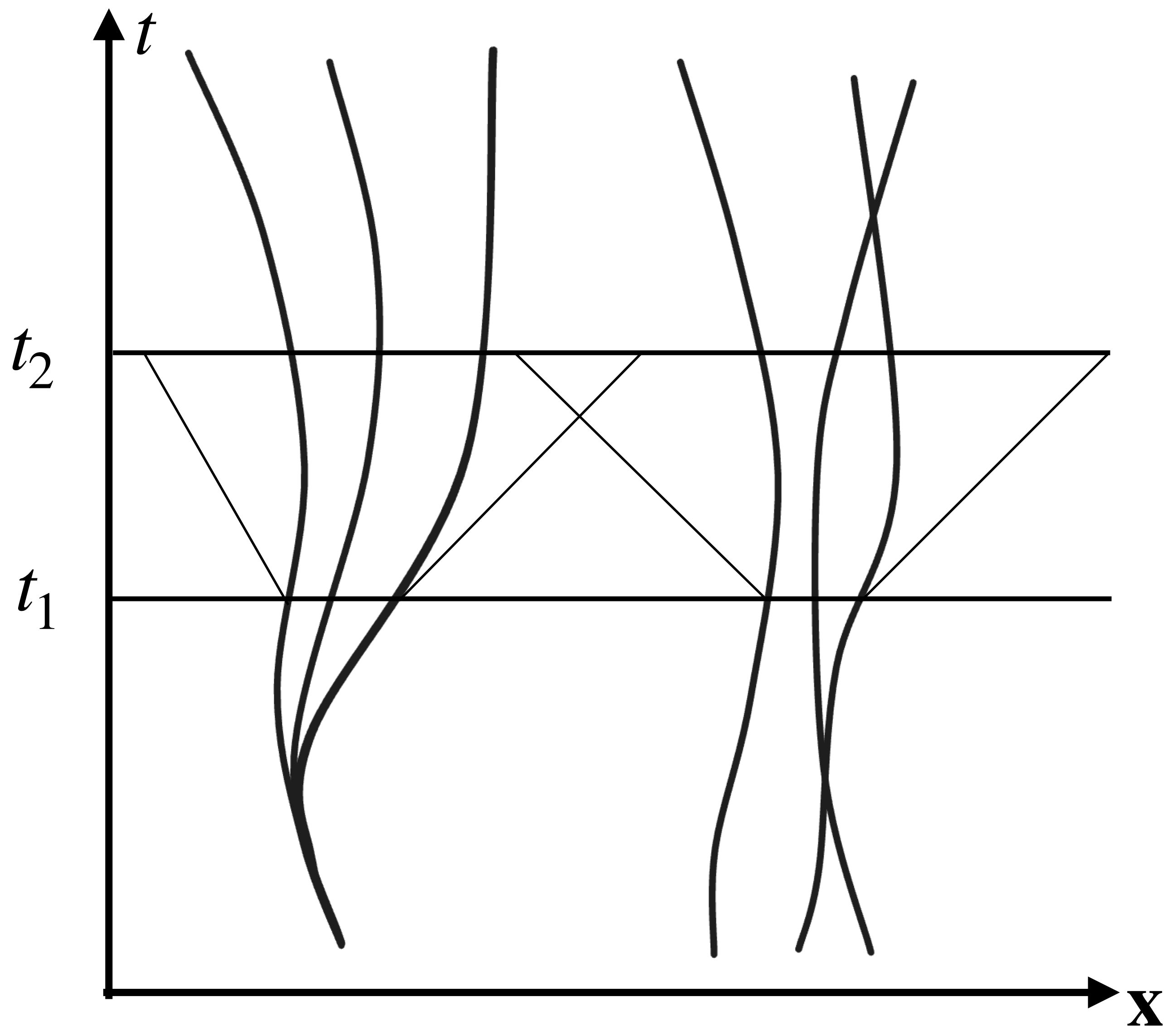
localisation only approximate

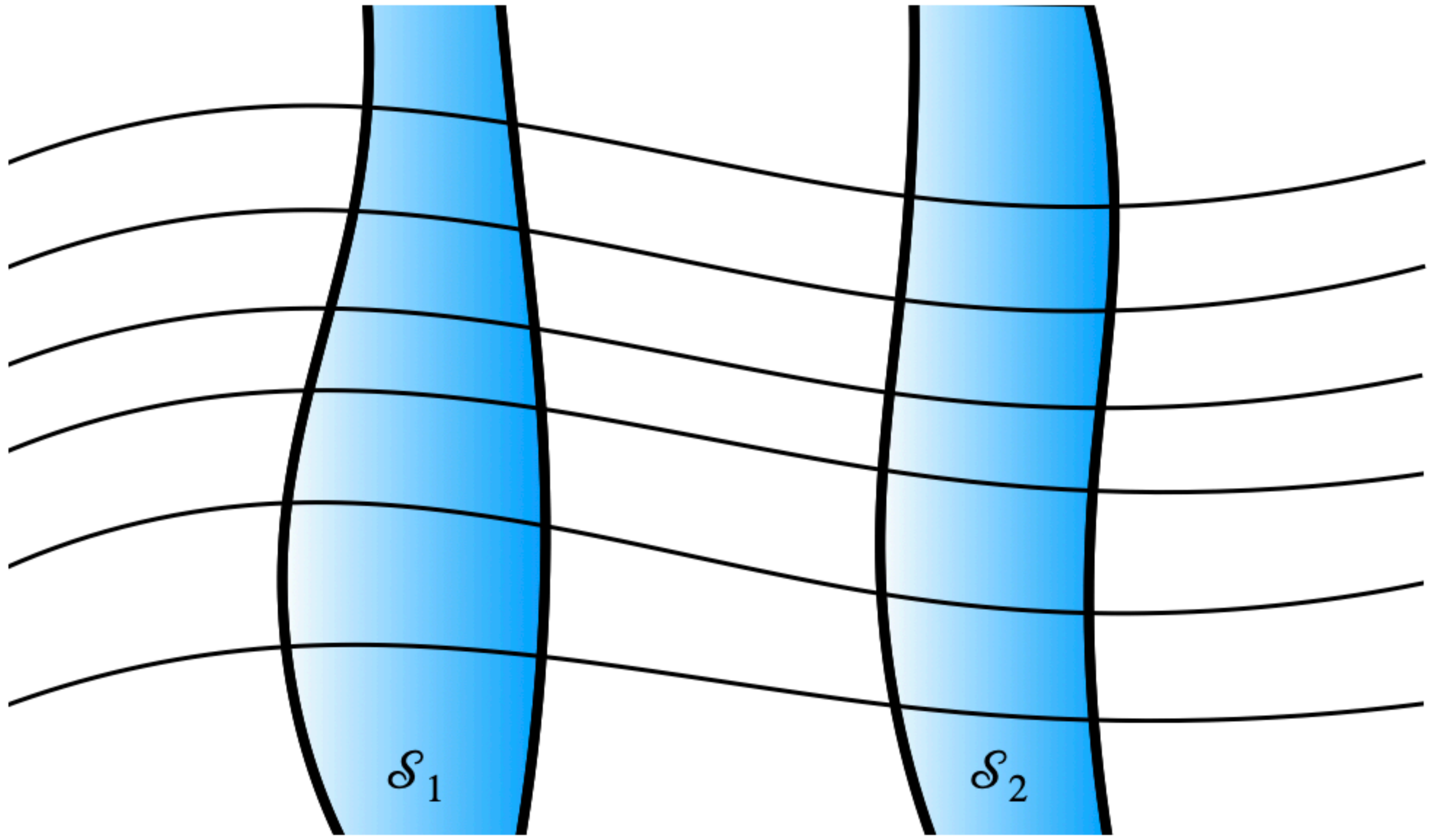
summary

mediation in QFT

$$\hat{H}(t) = \hat{H}_A(t) + \hat{H}_B(t) + \hat{H}_\phi + \hat{H}_{\text{int}}$$

$$\mathscr{L} = \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_\varphi + \mathscr{L}_{1\varphi} + \mathscr{L}_{2\varphi}$$





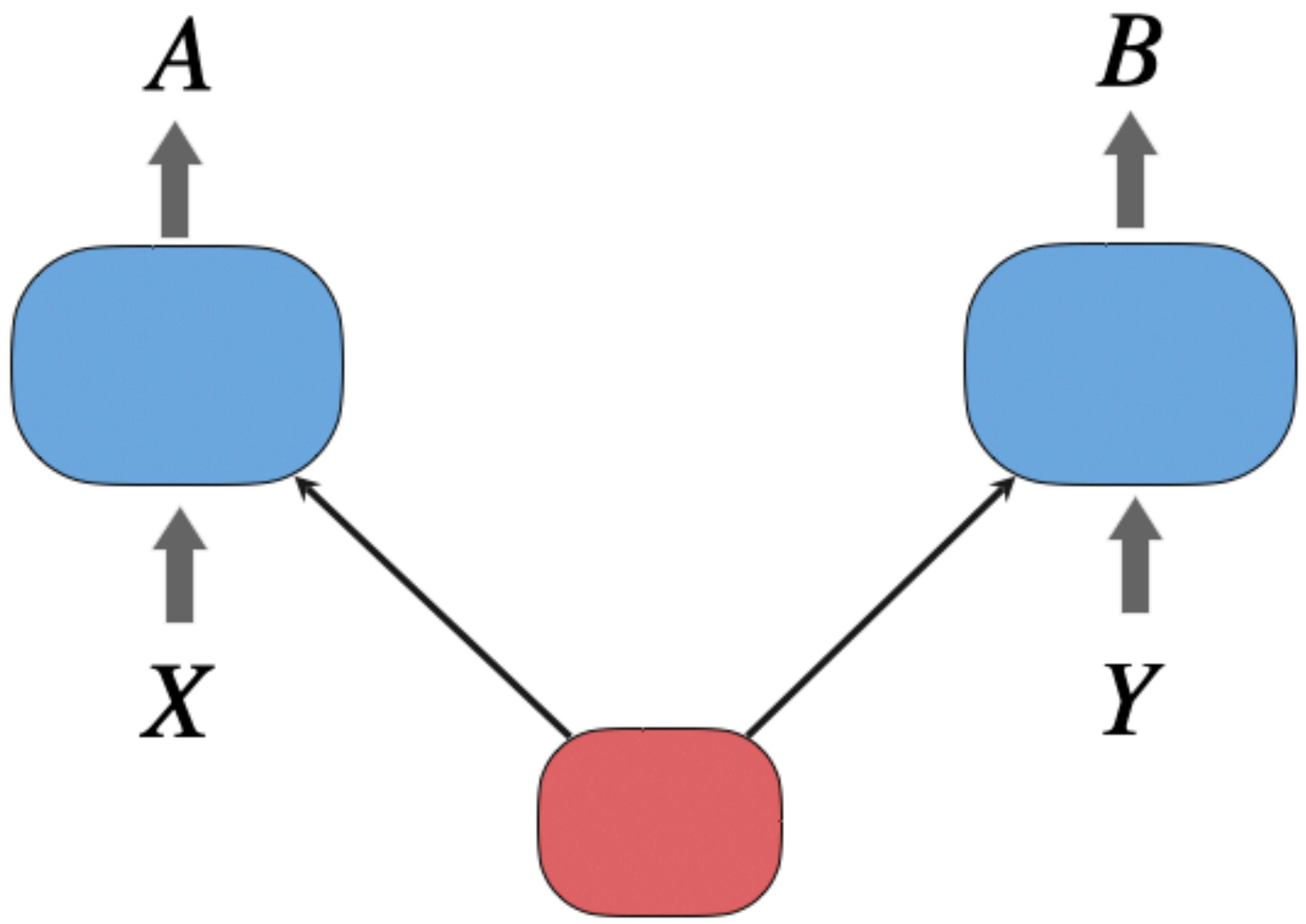
- **system-local coupling to *relativistic* field not sufficient to ensure mediation**
- **need assumptions on the states**
- **only approximate!**

no-go theorems?

no-go theorems

Bell 1976

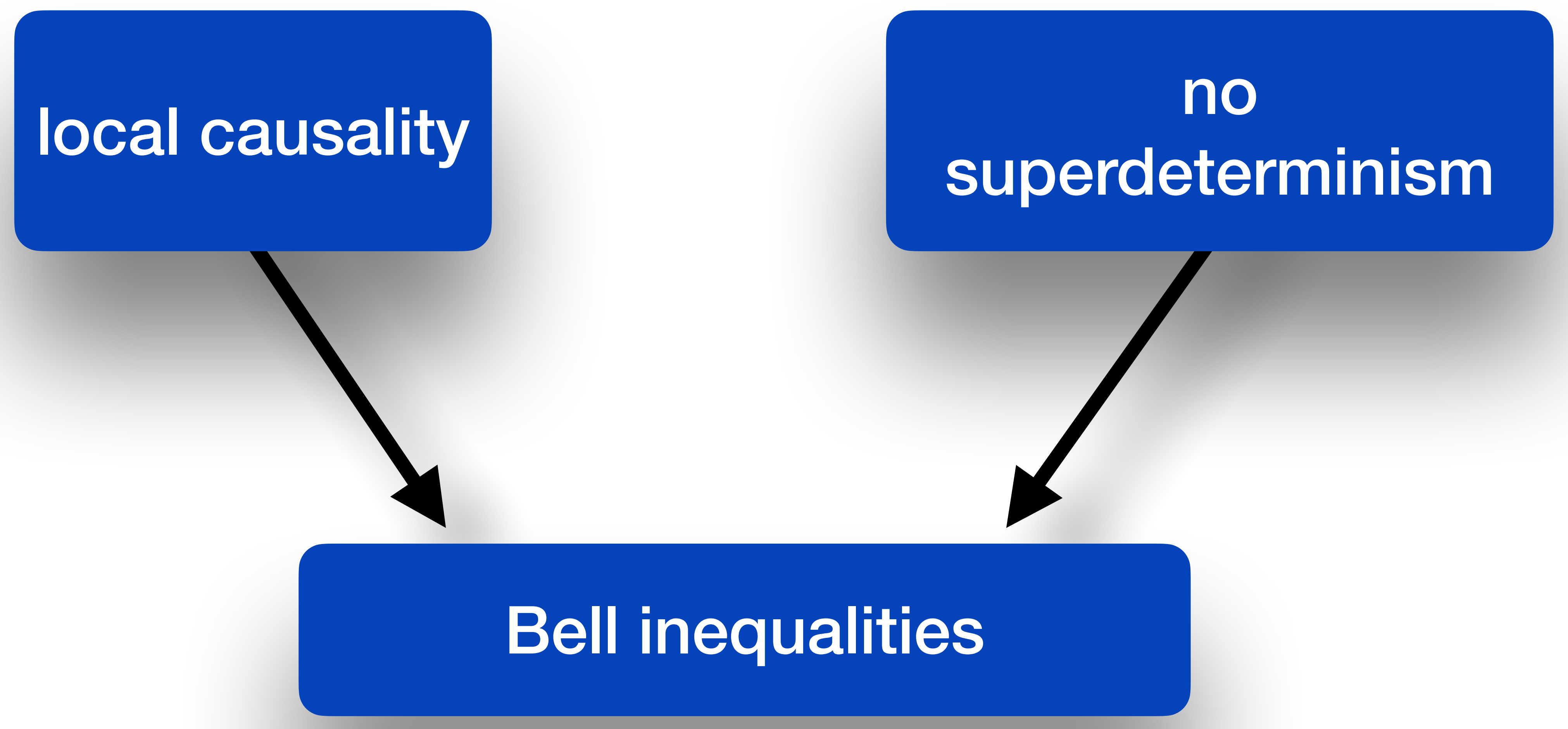
$$f(ab|xy) = \sum_{\lambda} p(ab|xy\lambda)p(\lambda|xy)$$



$$p(a|xy\lambda) = p(a|x\lambda)$$

$$p(b|xy\lambda) = p(b|y\lambda)$$

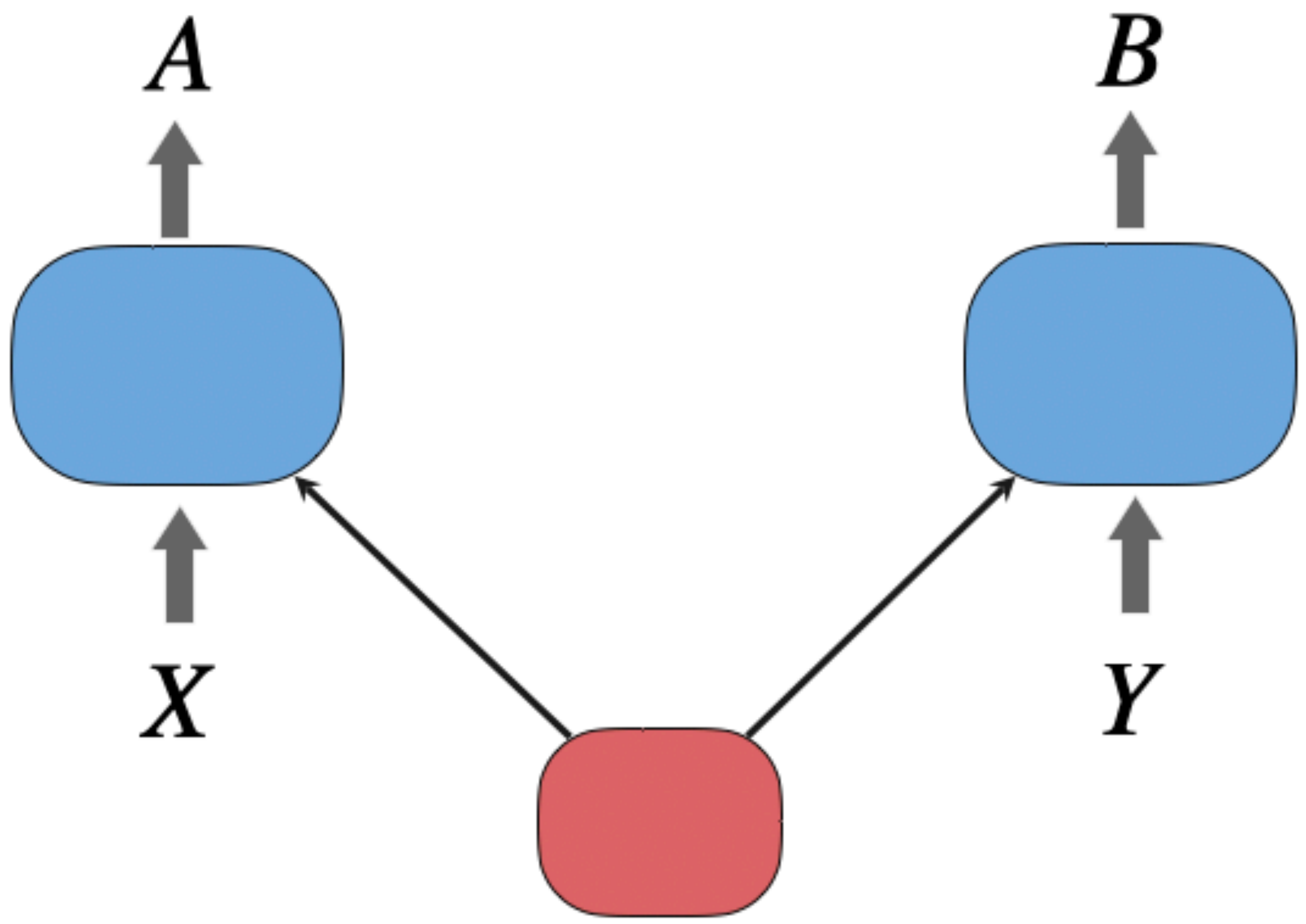
$$p(xy|\lambda) = p(xy)$$



no-go theorems

Bell 1976

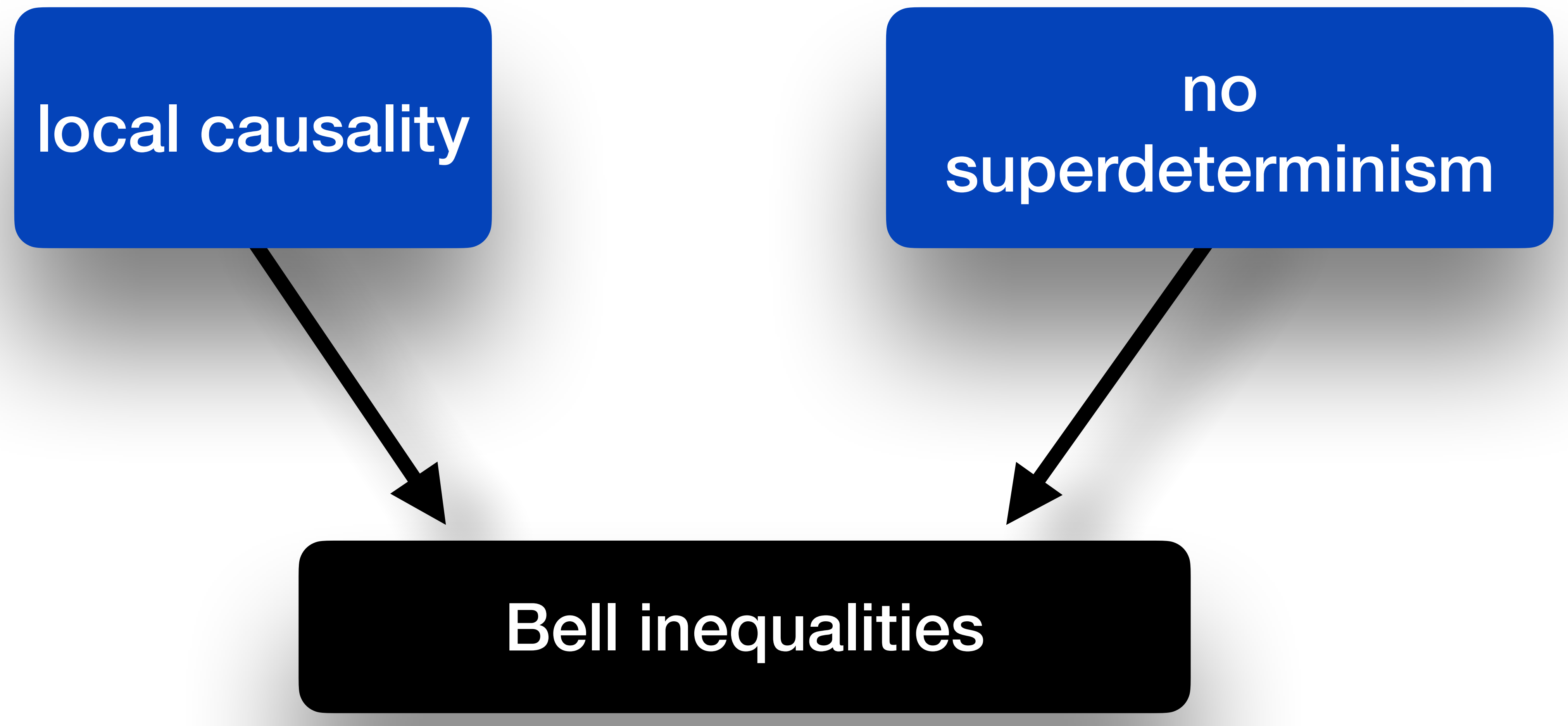
$$f(ab\,|\,xy) = \sum_{\lambda} p(ab\,|\,xy\lambda)p(\lambda\,|\,xy)$$



$$p(a\,|\,xy\lambda) = p(a\,|\,x\lambda)$$

$$p(b\,|\,xy\lambda) = p(b\,|\,y\lambda)$$

$$p(xy\,|\,\lambda) = p(xy)$$



experimental metaphysics

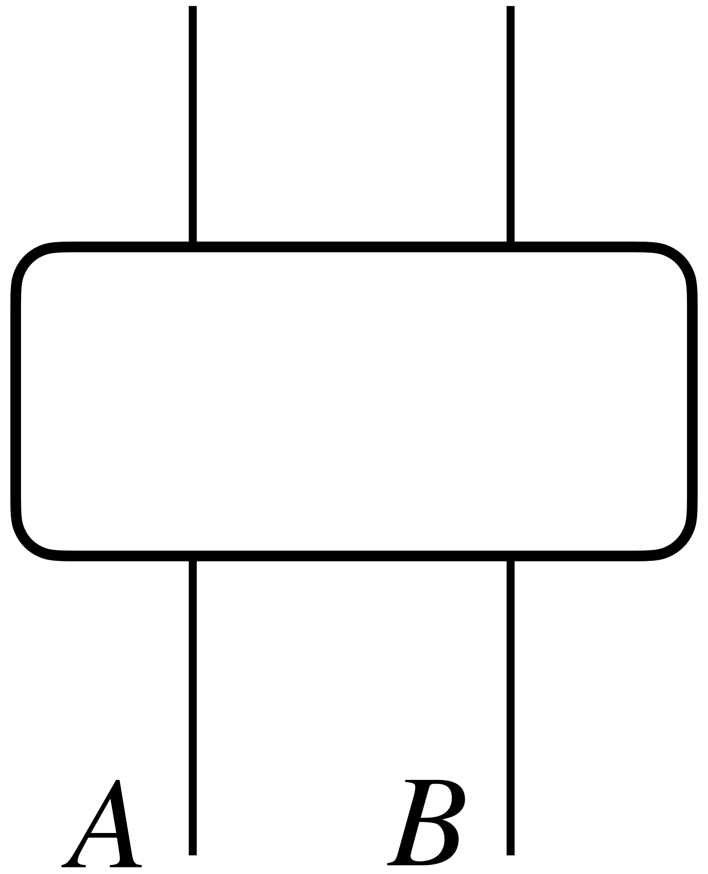
a *new way* of doing science:

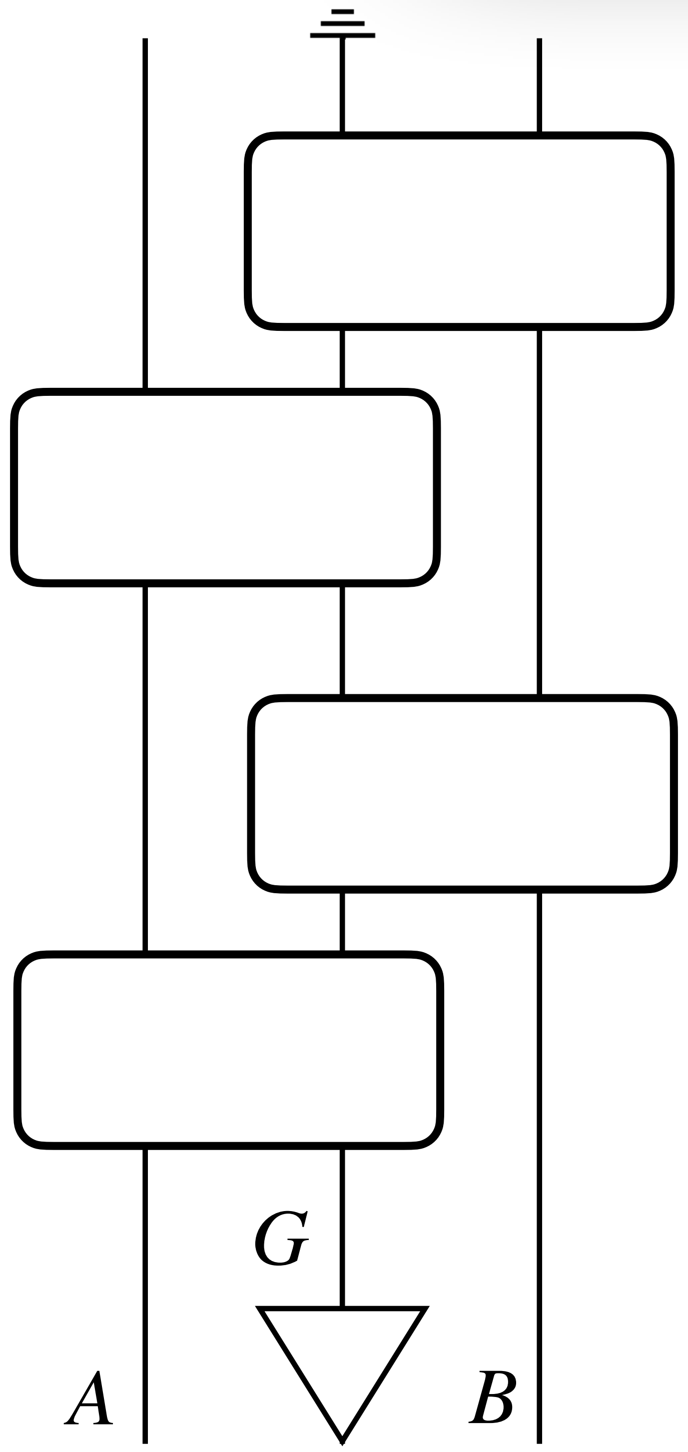
- **define a *space* T *of theories***
- **define a *subspace* $\mathsf{T}_{\mathrm{A}} \subset \mathsf{T}$ of theories based on set of assumptions A**
- **derive *experimental predictions* P_{A} for all theories $\mathsf{t} \in \mathsf{T}_{\mathrm{A}}$**
- **if experiment does not conform to P_{A}, rule out T_{A} and A**

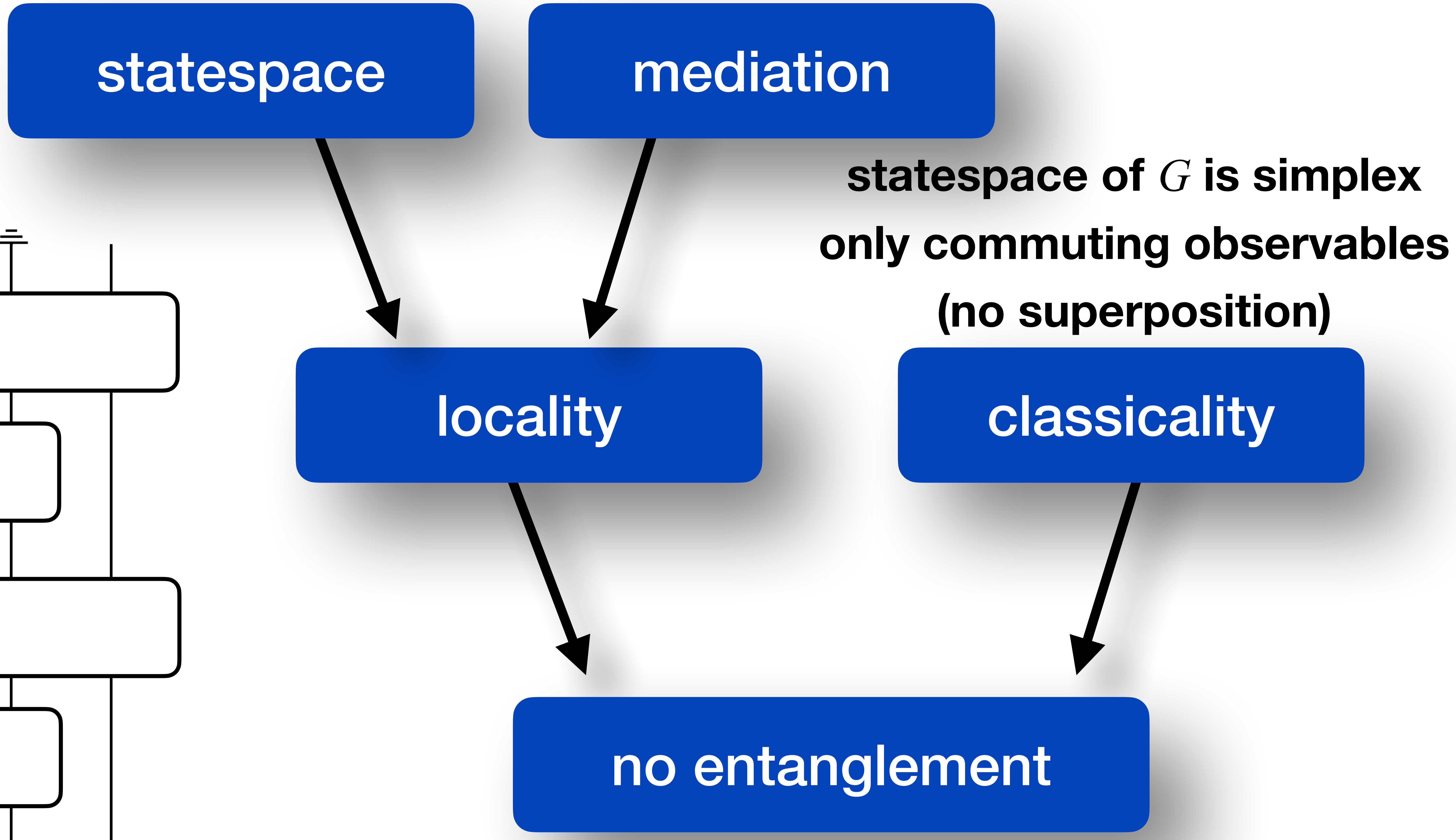
(how naturally) do our theories fit into T ?

how much do we care for the assumptions A?

GIE no-go theorems

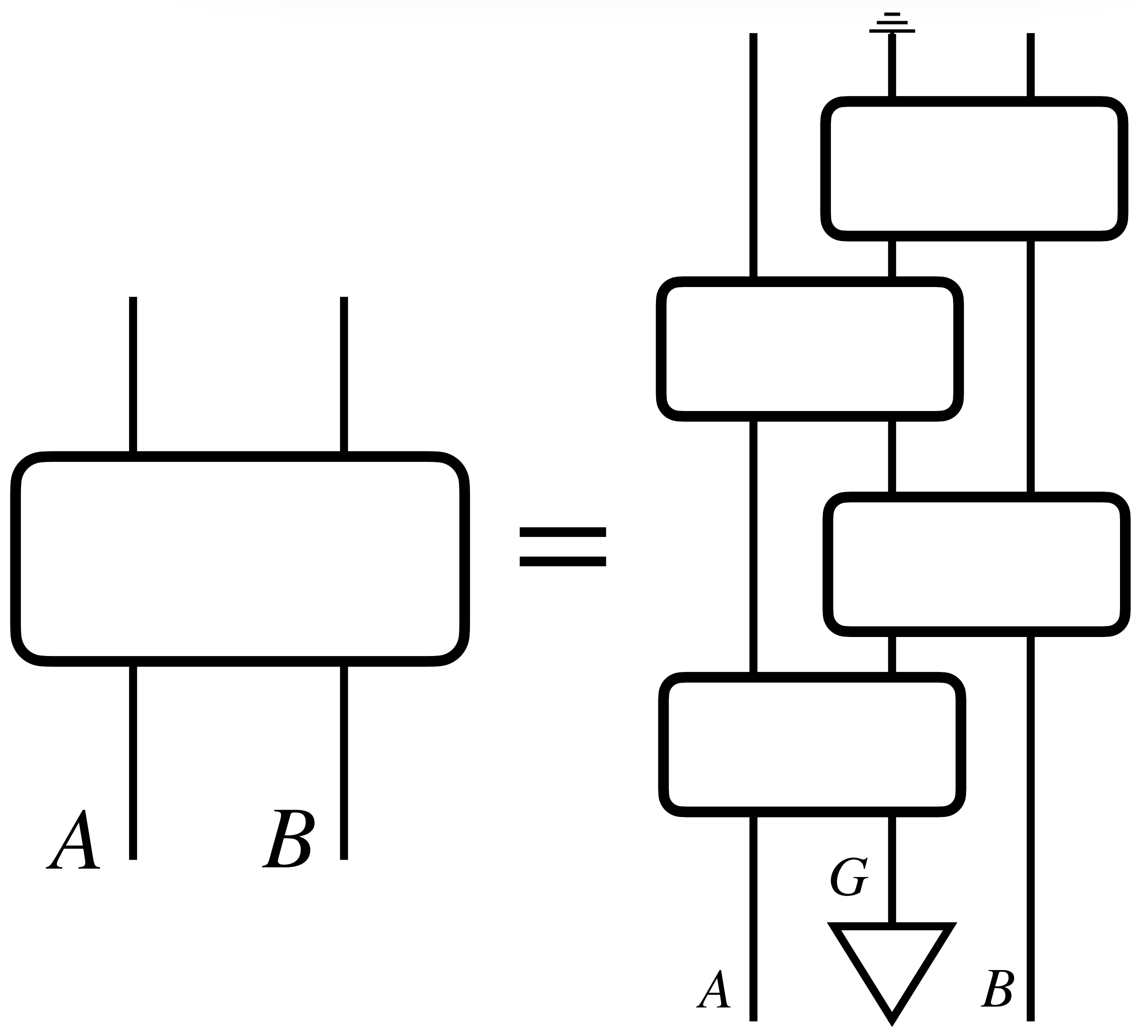


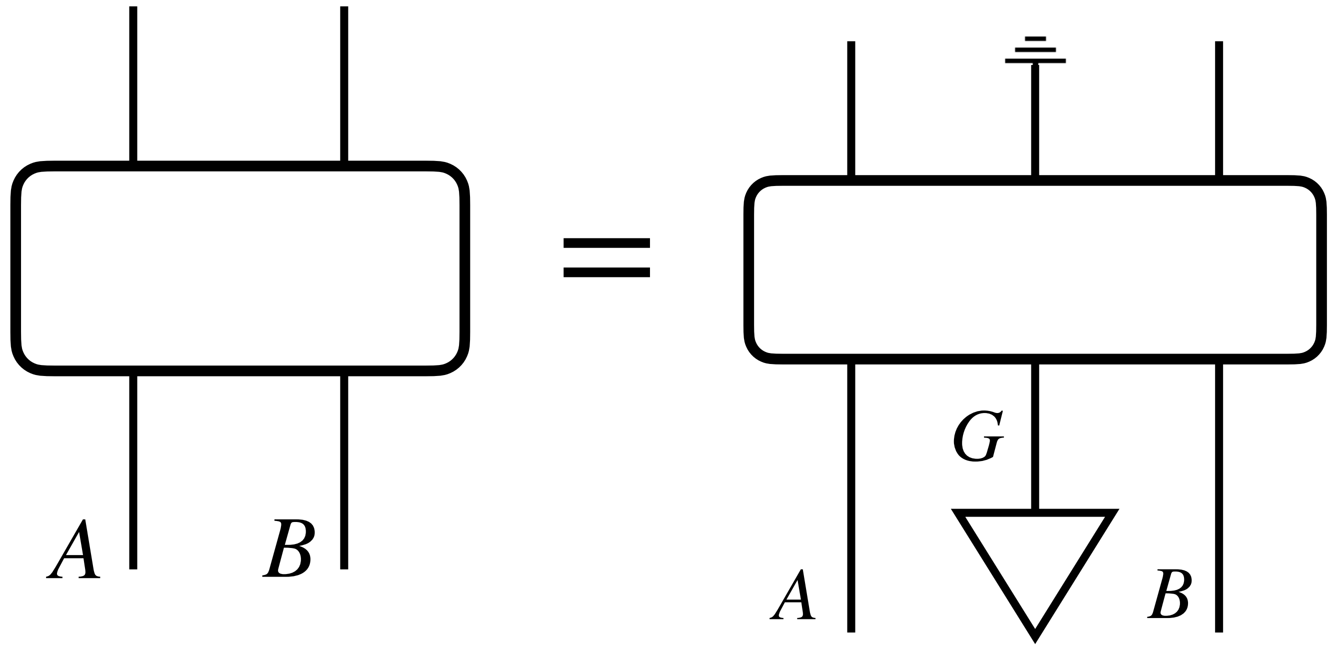


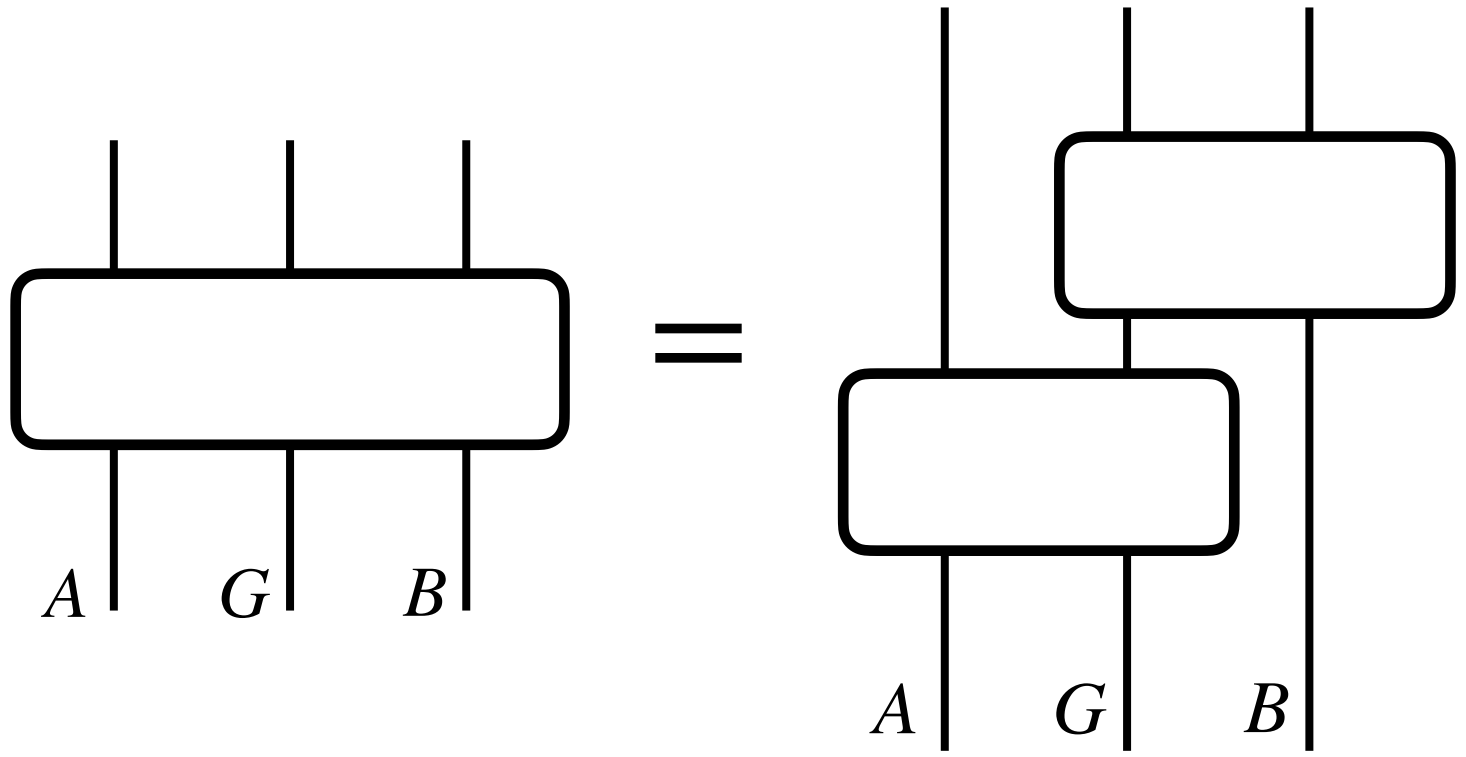


GIE no-go theorems

locality = **statespace** + **mediation**

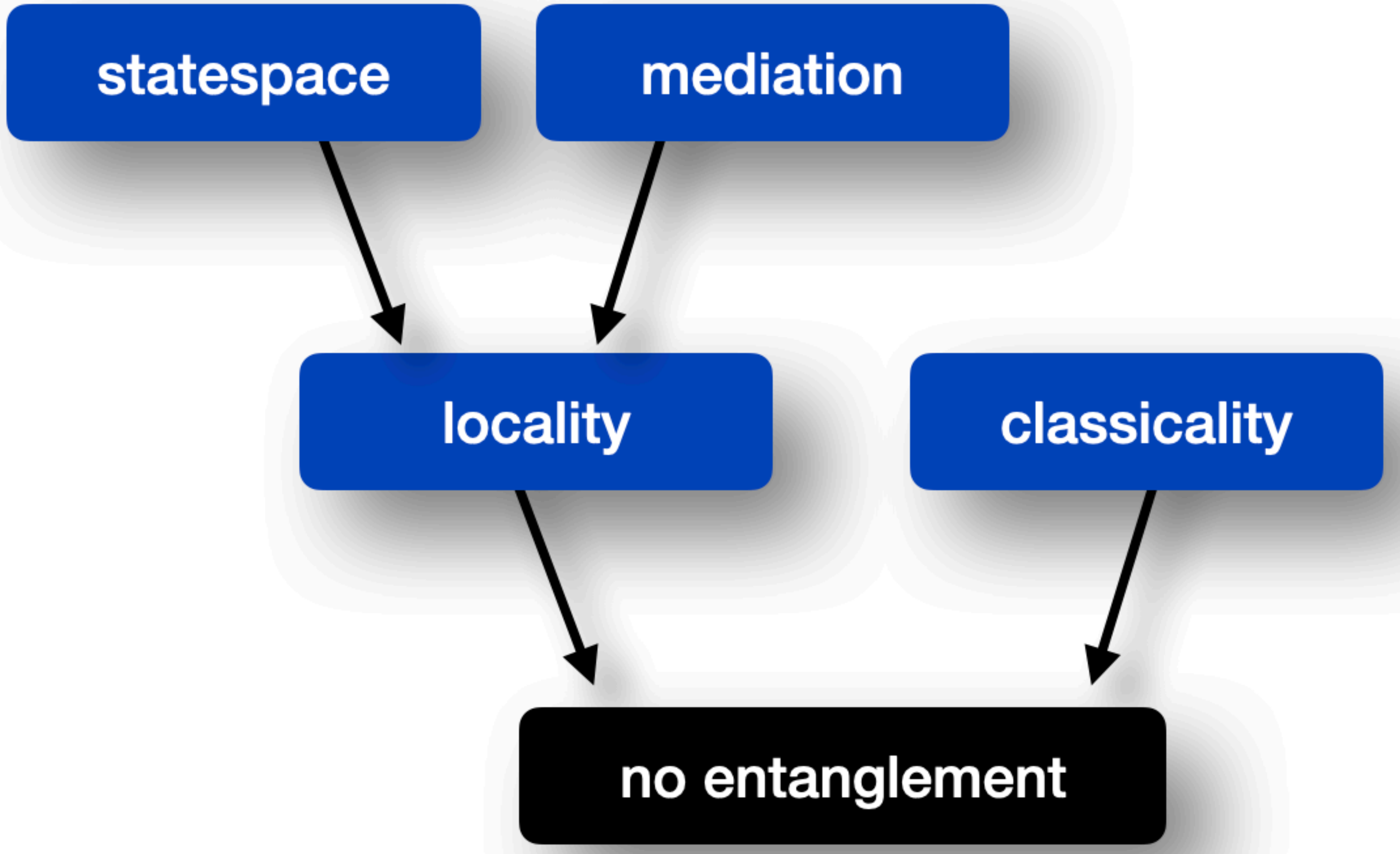






no-go theorems?

if we observe GIE



observing entanglement is not enough to rule out theories with classical gravity!

locality **assumption is not that natural (unlike Bell's local causality assumption)**

do we learn much doing the experiment?

YES! ***maybe?***

do the "old" kind of science: compare ***candidate*** **theories with experiment**

experimental predictions

theory	GIE?	assumption dropped	good candidate?
Newtonian QM	✅	statespace	❌ (GW)
semiclassical GR	❌ (?)	?	❌ (inconsistent)
LinQG (Lorenz gauge)	✅	classicality, statespace (?)	✅
LinQG (radiation gauge)	✅	statespace	✅
hybrid models	depends	mediation (?)	depends

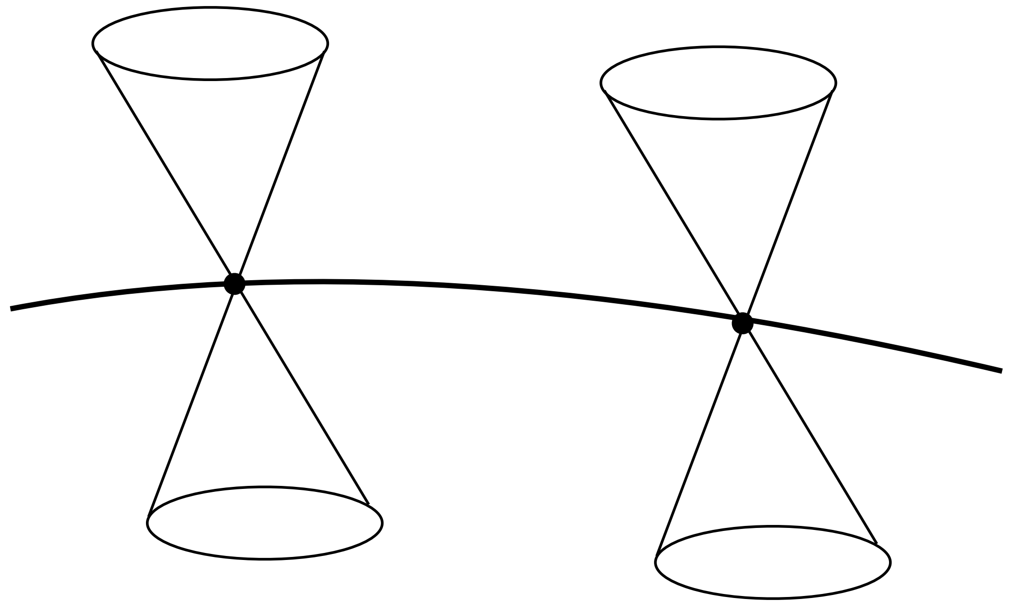
need to test *quantitative predictions of the different theories*

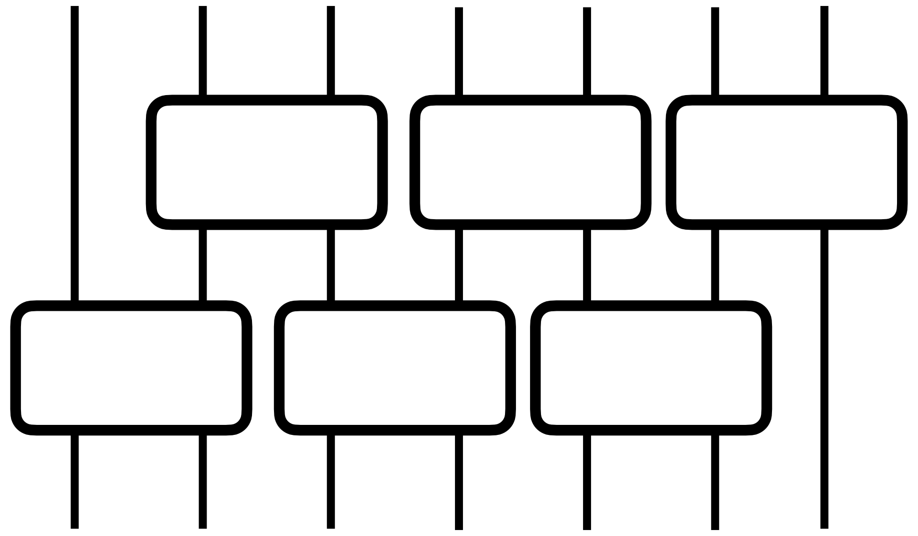
thank you!

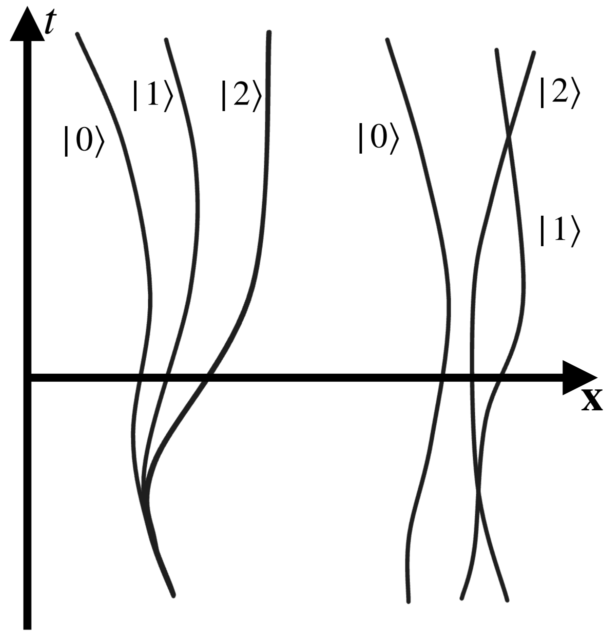
conclusion

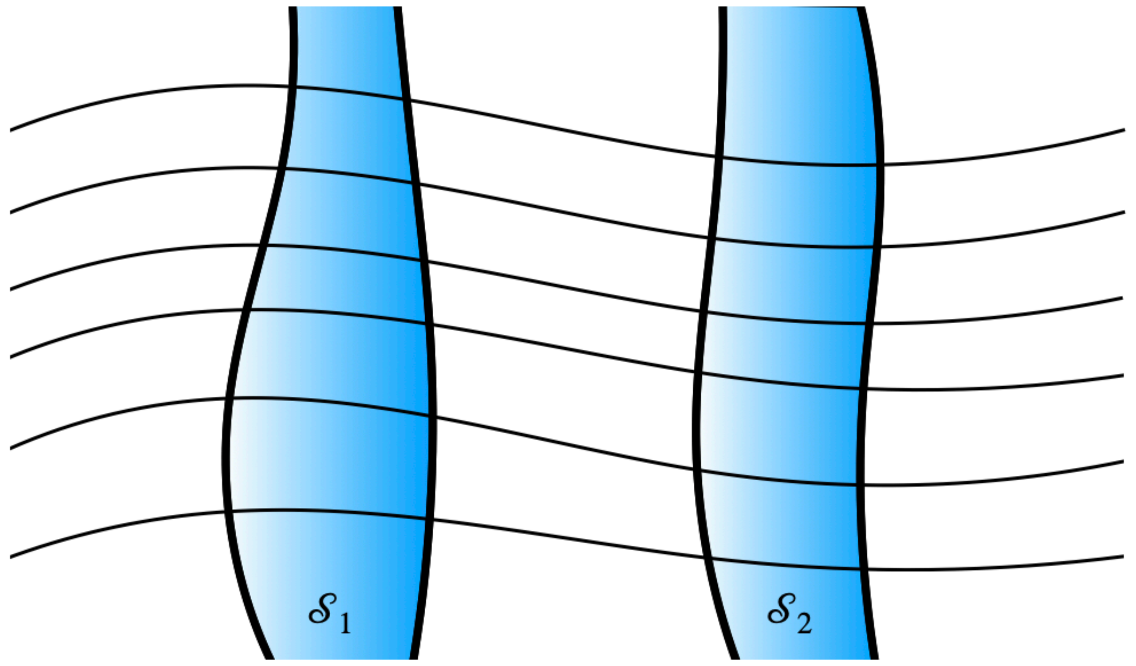
summary

- two notions of locality from different fields
- can obtain mediation from relativistic locality in QFT, *but only approximately*
- circuit locality seems not fundamental + gauge dependent
- implications for GIE no-gos









$$H|\psi_1\rangle \approx \left(H_1 + H_2 + \frac{q_1 q_2}{|\mathbf{x}_1 - \mathbf{x}_2|}\right)|\psi_1\rangle$$