When does relativistic locality imply subsystem locality?

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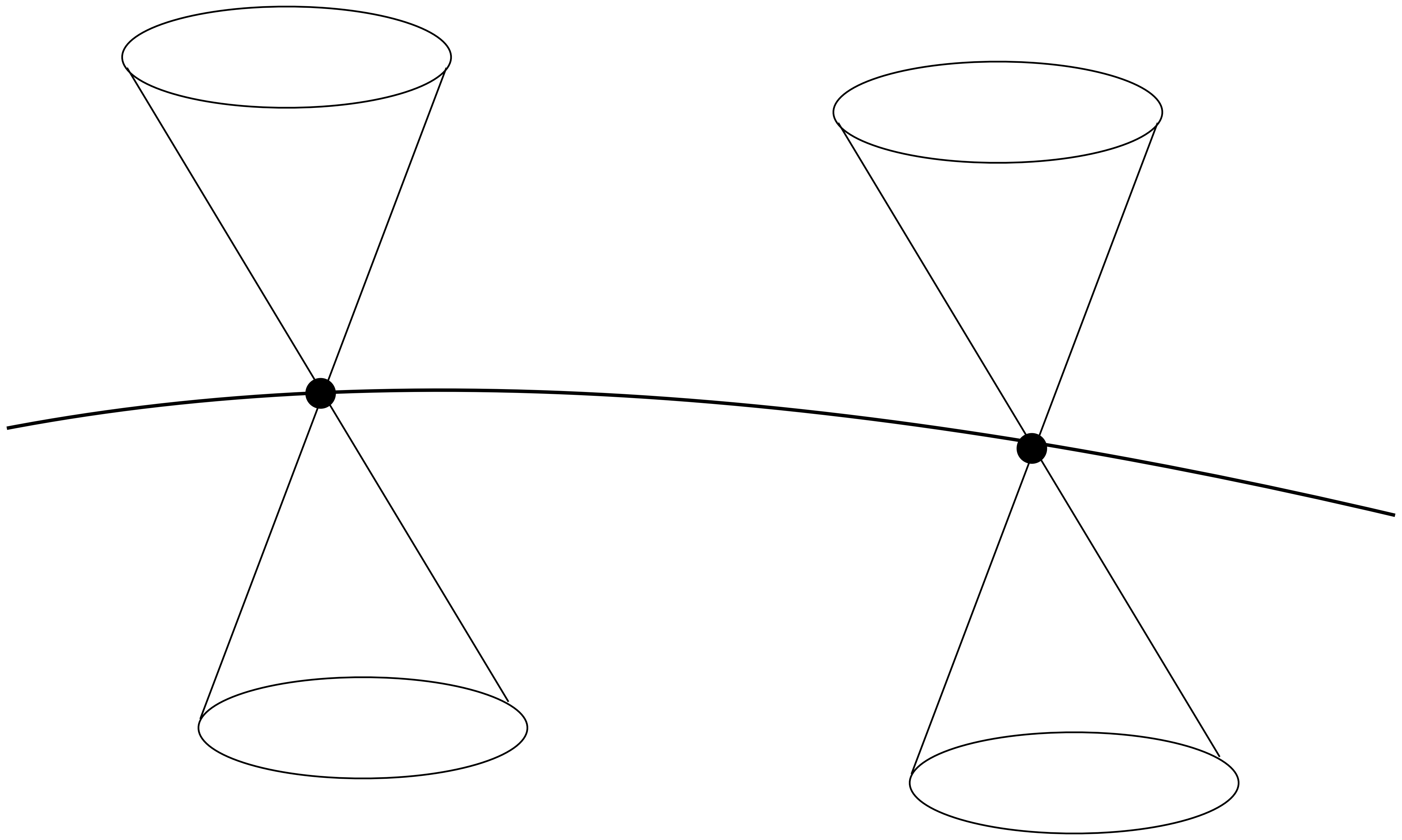
2023-11-10

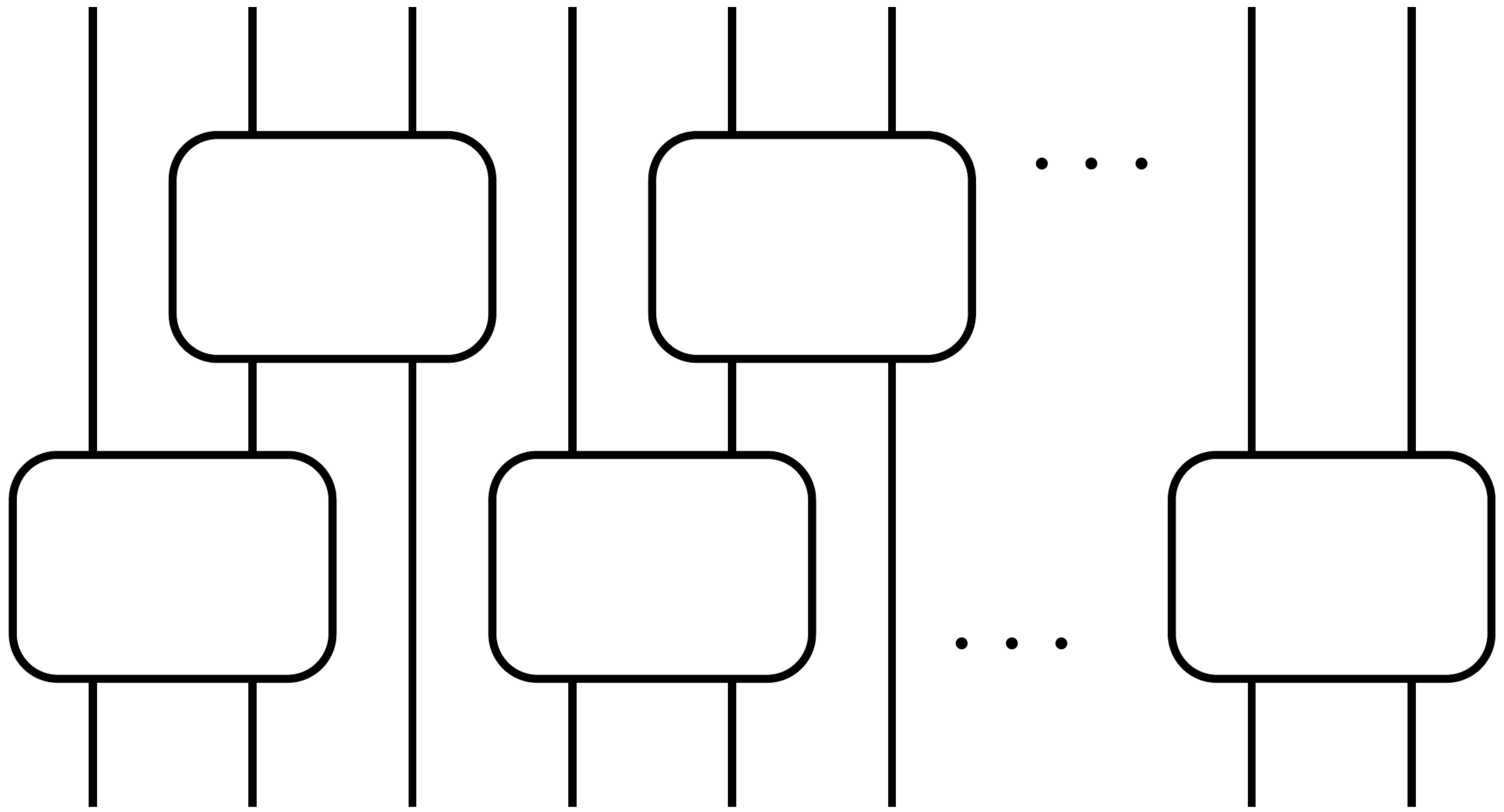
two notions of locality

Relativistic



Spacetime regions

Quantum Information



Systems

low energy quantum gravity

If we detect gravity mediated entanglement, then gravity cannot be both:

classical **local**

Spin Entanglement Witness for Quantum Gravity

Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A. Geraci, Peter F. Barker, M. S. Kim, and Gerard Milburn

Phys. Rev. Lett. **119**, 240401 – Published 13 December 2017

QM

Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity

C. Marletto and V. Vedral

Phys. Rev. Lett. **119**, 240402 – Published 13 December 2017

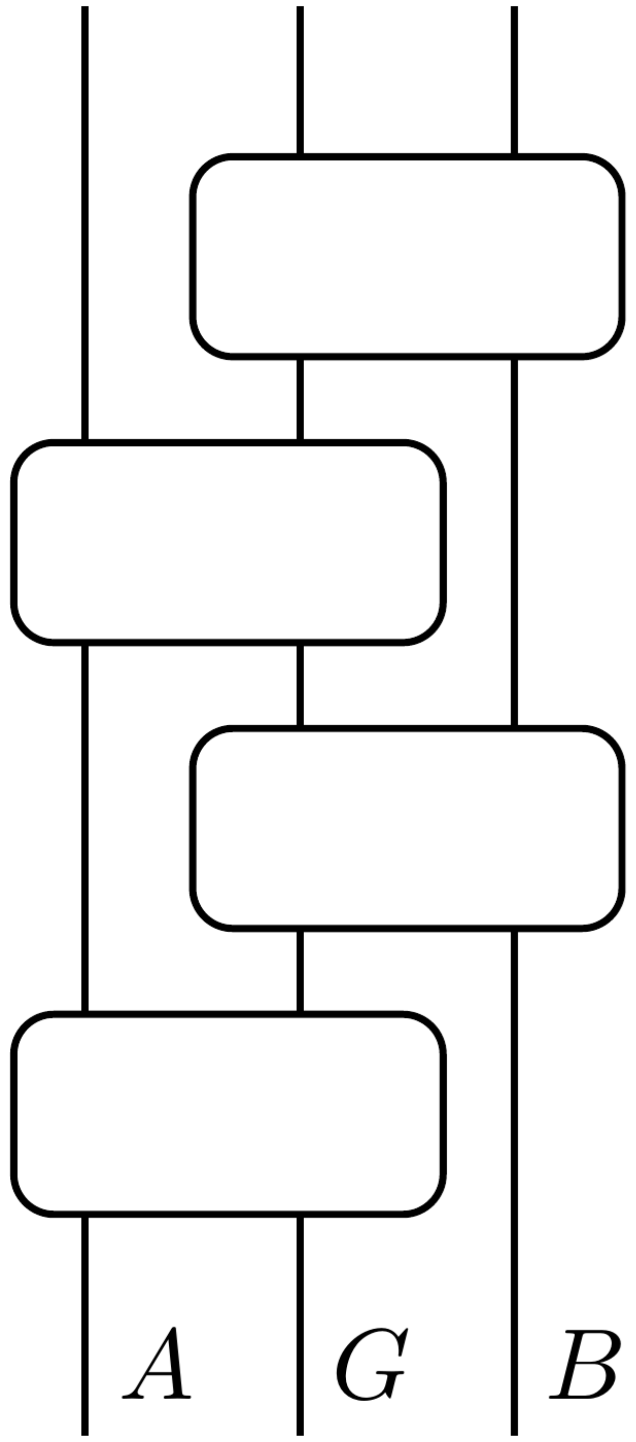
constructor theory

A no-go theorem on the nature of the gravitational field beyond quantum theory

Thomas D. Galley[1], Flaminia Giacomini[1], and John H. Selby[2]

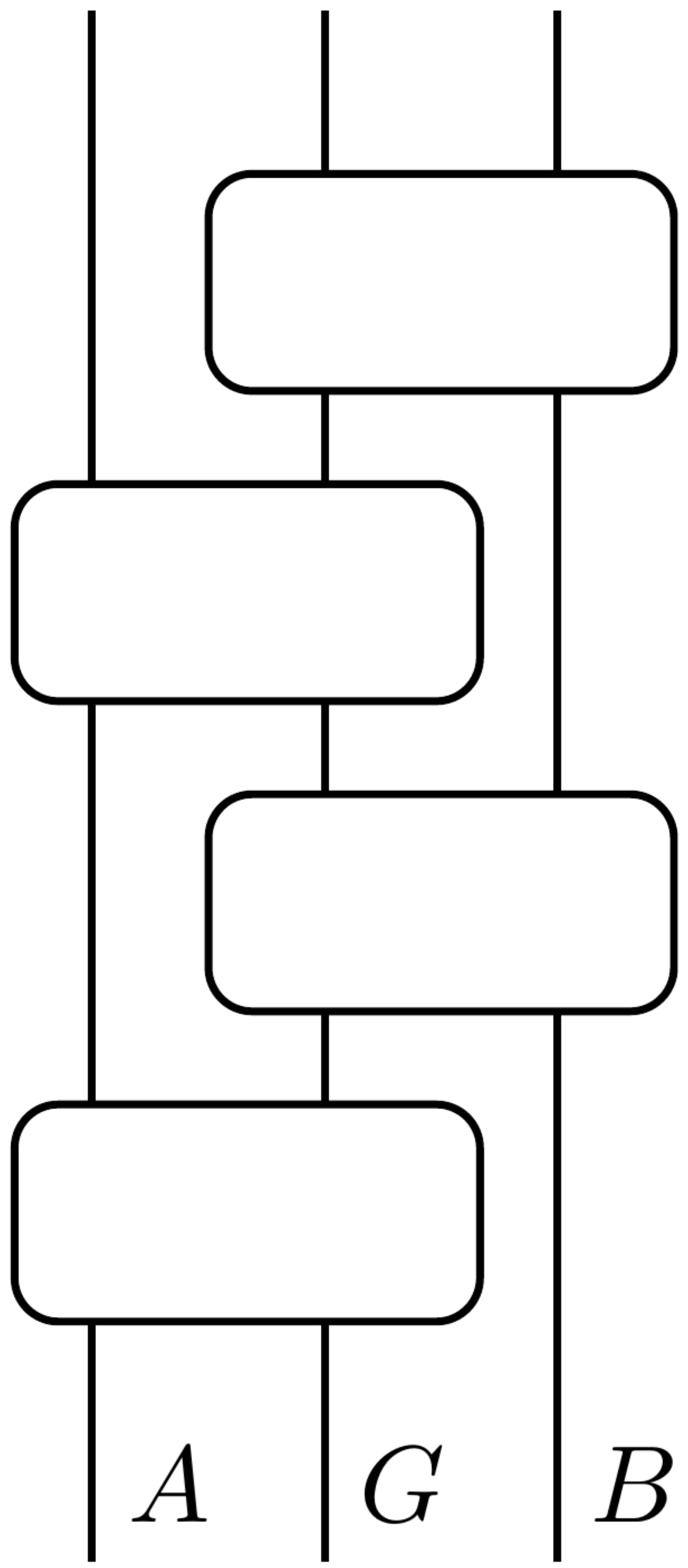
Published:	2022-08-17, volume 6, page 779
Eprint:	arXiv:2012.01441v7
Doi:	https://doi.org/10.22331/q-2022-08-17-779
Citation:	Quantum 6, 779 (2022).

GPTs



a foundation for subsystem locality?

- Subsystem locality is a often good operational assumption.
  - Axiomatically assumed in quantum foundations works
  - Motivated by relativistic intuition
- Does subsystem locality hold in QFT?
  - Does a field mediate interactions, in the QI sense?
  - To what extent does this property hold in nature?



plan

- **two false starts**
- **the scalar field theory case**
- **open questions**

two false starts

Suzuki-Trotter

$$H = H_A + H_B + H_C + H_{AC} + H_{BC}$$

Suzuki-Trotter

$$H = H_{AC} + H_{BC}$$

$$\implies U(t) = e^{-i(H_{AC}+H_{BC})t} \neq e^{-iH_{AC}t} e^{-iH_{BC}t}$$

$$= \lim_{n\to\infty} \left(e^{-iH_{AC}\,t/n} e^{-iH_{BC}\,t/n} \right)^n$$

Arbitrarily good approximation **but no input from relativity.**

QED in Coulomb gauge

$$H = H_1 + H_2 + H^{\text{rad}}_{A_\perp} + \frac{q_1 q_2}{|\mathbf{x}_1 - \mathbf{x}_2|} - \int \mathrm{d}^3\mathbf{x}\, A_\perp(\mathbf{x}) \cdot \big(J_1(\mathbf{x}) + J_2(\mathbf{x})\big)$$

If two particles are at rest, and there is no radiation, then

$$H \approx \frac{q_1 q_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

Not subsystem local!

massive scalar field

massive scalar field

a positive result

arXiv:2305.05645 (quant-ph)

[Submitted on 9 May 2023]

Relativistic locality can imply subsystem locality

Andrea Di Biagio, Richard Howl, Časlav Brukner, Carlo Rovelli, Marios Christodoulou

Concrete example:

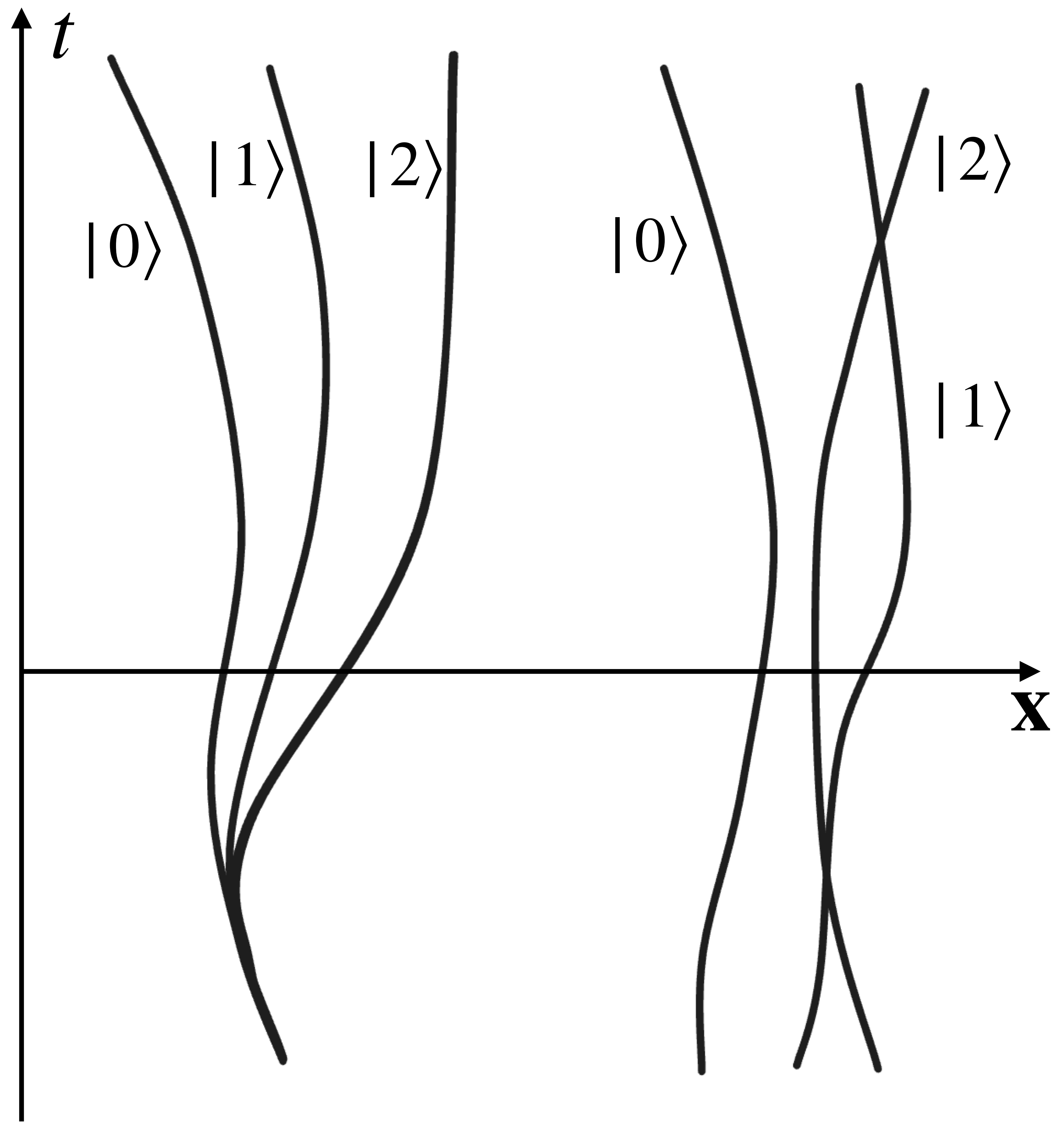
- **Two particles coupled to a massive scalar field, in a specific regime.**
- **Evolution is subsystem local, *up to some phases*.**
- **Microcausality ($[\hat{\phi}(x), \hat{\phi}(x')] = 0$ if x, x' spacelike) eliminates the phases.**

- **Relativistic locality implies subsystem locality.**

massive scalar field

three key assumptions

- **Support of the matter wavefunctions contained within two distinct spacetime regions.**
- **Matter in quantum-controlled superposition of semi-classical states.**



derivation sketch

derivation

setup

$$\hat{H}(t) = \hat{H}_A(t) + \hat{H}_B(t) + \hat{H}_0 + \hat{H}_{\text{int}}$$

quantum-controlled dynamics

$$\hat{H}_A(t) = \sum_r |r\rangle\langle r| \otimes \hat{H}_A^r(t)$$

kinetic field term

$$\hat{H}_0 = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}$$

local interaction

$$\hat{H}_{\text{int}} = \int \mathrm{d}^3\mathbf{x}\, \hat{\phi}(\mathbf{x}) \big(\hat{\mu}_A(\mathbf{x}) + \hat{\mu}_B(\mathbf{x})\big)$$

derivation

qudit-controlled dynamics

No back action on the qudits + matter in superposition of pointer states:

$$|\Psi(t)\rangle = \sum_{rs} c_{rs} |rs\rangle |\psi_A^r(t)\rangle |\psi_B^s(t)\rangle |\phi^{rs}(t)\rangle$$

Particles: $$\frac{\mathrm{d}}{\mathrm{d}t} |\psi_A^r(t)\rangle = -i\hat{H}_A^r(t) |\psi_A^r(t)\rangle$$

Field: $$\frac{\mathrm{d}}{\mathrm{d}t} |\phi^{rs}(t)\rangle = -i\left(\hat{H}_0 + \hat{H}_{\mathrm{int}}^{rs}(t)\right) |\phi^{rs}(t)\rangle \qquad \hat{H}_{\mathrm{int}}^{rs}(t) = \langle \psi_A^r(t)\psi_B^s(t) | \hat{H}_{\mathrm{int}} | \psi_A^r(t)\psi_B^s(t)\rangle$$

Evolution of the whole system: $$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}_A^r \otimes \hat{U}_B^s \otimes \hat{U}_\phi^{rs}$$

derivation

condition for subsystem locality

$$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}_A^r \otimes \hat{U}_B^s \otimes \hat{U}_\phi^{rs}$$ **is not subsystem local**

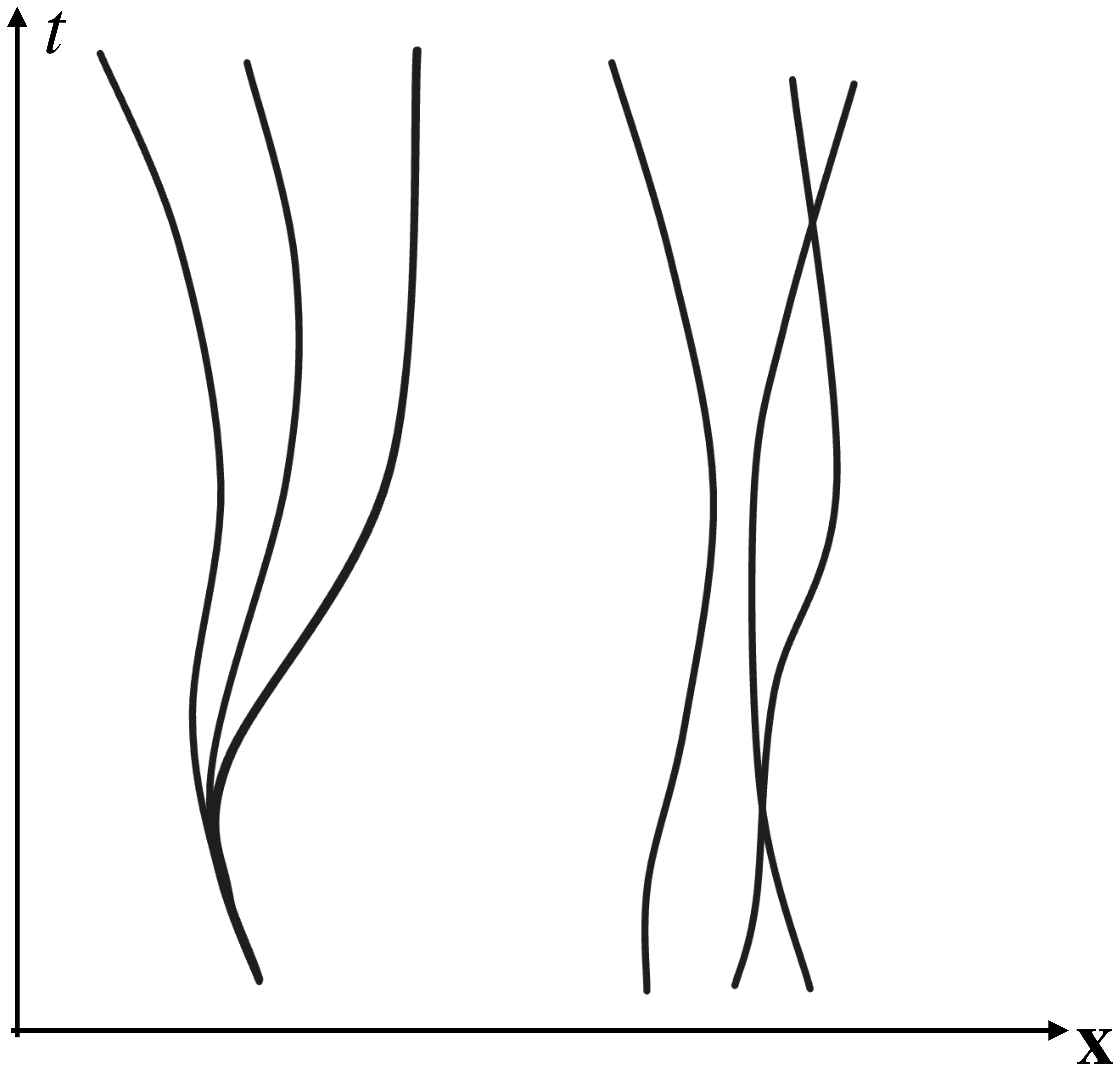
but if we had $\forall rs \; : \; \hat{U}_\phi^{rs} = \hat{U}_\phi^r \circ \hat{U}_\phi^s$ **then it would be:**

$$\hat{U} = \left(\sum_s |s\rangle\langle s| \otimes \hat{U}_B^s \otimes \hat{U}_\phi^s \right) \circ \left(\sum_r |r\rangle\langle r| \otimes \hat{U}_A^r \otimes \hat{U}_\phi^r \right)$$

derivation

evolution of the field

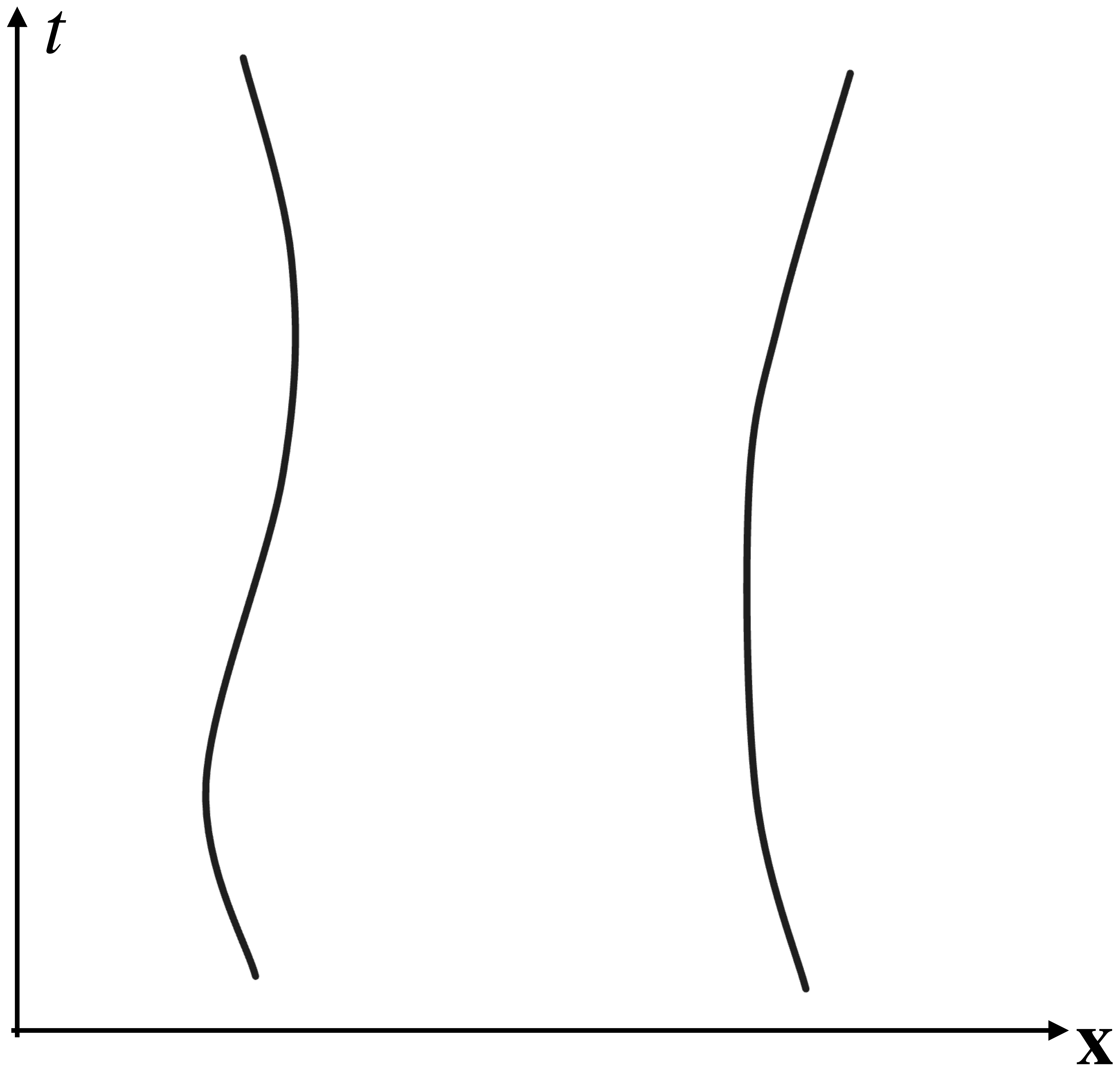
$$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}_A^r \otimes \hat{U}_B^s \otimes \hat{U}_\phi^{rs}$$



$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}_\phi^{rs}(t) = -i\hat{H}^{rs}(t)\hat{U}_\phi^{rs}(t)$$

derivation

evolution of the field



quantum field with classical source!

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}^{rs}_{\phi}(t) = -\,i\hat{H}^{rs}(t)\hat{U}^{rs}_{\phi}(t)$$

$$\hat{H}^{rs}(t) = \hat{H}_0 + \langle \psi^r_A(t)\psi^s_B(t)\,|\,\hat{H}_{\mathrm{int}}\,|\,\psi^r_A(t)\psi^s_B(t)\rangle$$

exact solution

$$\hat{U}^{rs}_{\phi} = e^{i\Omega^{rs}}\hat{D}^{rs}e^{-i\hat{H}_0(t_2-t_1)}$$

derivation

subsystem locality?

$$\hat{U}_\phi^{rs} \quad = \quad e^{i\Omega^{rs}} \hat{D}^{rs} e^{-i\hat{H}_0(t_2 - t_1)} \quad = \quad e^{i\tilde{\Omega}^{rs}} \hat{U}_\phi^r \hat{U}_\phi^s e^{-i\hat{H}_0(t_2 - t_1)}$$

full evolution:

$$\hat{U} = \sum_{sr} e^{i\tilde{\Omega}^{rs}} \left(|s\rangle\langle s| \, \hat{U}_B^s \otimes \hat{U}_\phi^s \right) \circ \left(|r\rangle\langle r| \, \hat{U}_A^r \otimes \hat{U}_\phi^r \right) \circ e^{-i\hat{H}_0(t_2 - t_1)}$$

almost subsystem local!

derivation

the phase

$$\tilde{\Omega}^{rs} = -i \iint_{t_1}^{t_2} \mathrm{d}t \mathrm{d}t' \iint \mathrm{d}^3\mathbf{x} \mathrm{d}^3\mathbf{x}' \, \mu_A^r(t,\mathbf{x}) \mu_B^s(t',\mathbf{x}') [\hat{\phi}_I(t,\mathbf{x}), \hat{\phi}_I(t',\mathbf{x}')]$$

$$-i \int_{t_1}^{t_2} \mathrm{d}t \int_{t_1}^{t} \mathrm{d}t' \iint \mathrm{d}^3\mathbf{x} \mathrm{d}^3\mathbf{x}' \Big(\mu_A^r(t,\mathbf{x}) \mu_B^s(t',\mathbf{x}') + \mu_B^r(t,\mathbf{x}) \mu_A^s(t',\mathbf{x}') \Big) [\hat{\phi}_I(t,\mathbf{x}), \hat{\phi}_I(t',\mathbf{x}')]$$

derivation

relativistic locality

$$\tilde{\Omega}^{rs} = -i \iint_{t_1}^{t_2} \mathrm{d}^4x \mathrm{d}^4x \, \mu_A^r(x) \mu_B^s(x') [\hat{\phi}_I(x), \hat{\phi}_I(x')] - \cdots$$

Microcausality: $$[\hat{\phi}_I(x), \hat{\phi}_I(x')] = 0$$

if x and x' are spacelike

derivation

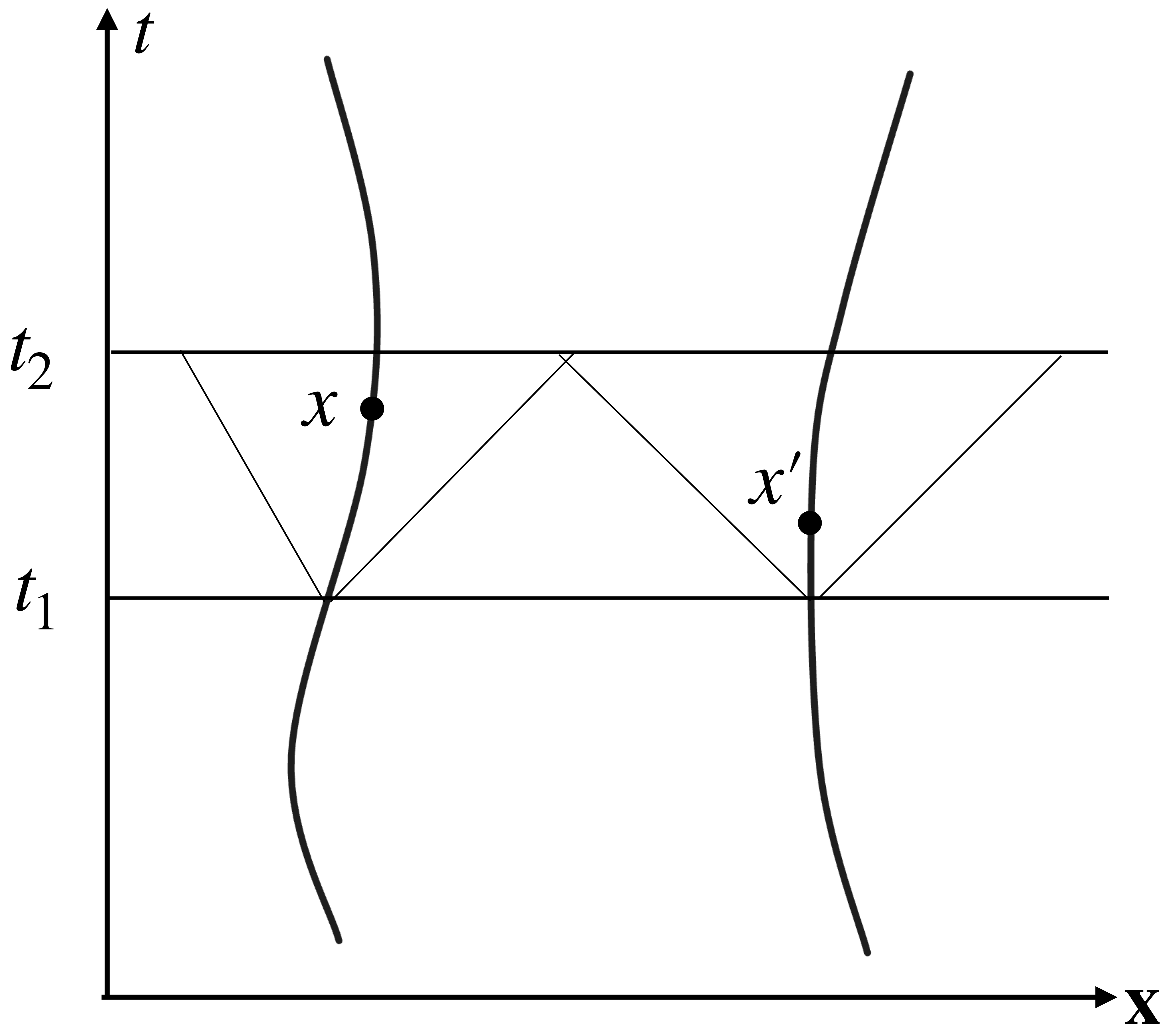
relativistic locality

$$\tilde{\Omega}^{rs} = -\,i \iint_{t_1}^{t_2} \mathrm{d}^4x\mathrm{d}^4x\, \mu_A^r(x)\mu_B^s(x')[\hat{\phi}_I(x), \hat{\phi}_I(x')] - \cdots$$

if $\operatorname{supp}\mu_A^r, \quad \operatorname{supp}\mu_B^s$

are spacelike

then $\tilde{\Omega}^{rs} = 0$



derivation

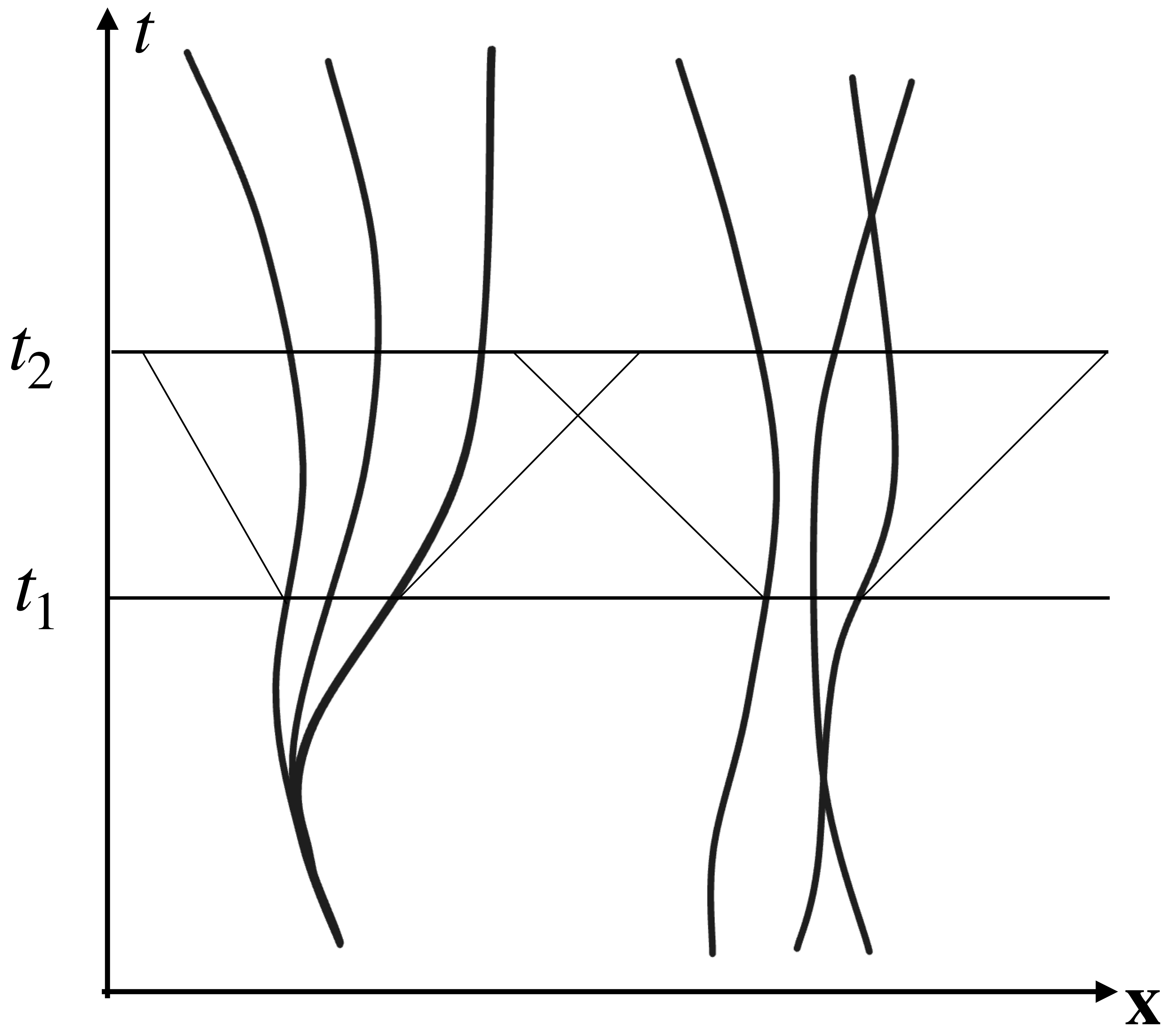
relativistic locality

$$\tilde{\Omega}^{rs} = -i \iint_{t_1}^{t_2} \mathrm{d}^4x \mathrm{d}^4x \, \mu_A^r(x) \mu_B^s(x') [\hat{\phi}_I(x), \hat{\phi}_I(x')] - \cdots$$

if $\operatorname{supp} \mu_A^r, \quad \operatorname{supp} \mu_B^s$

are spacelike $\forall rs$

then $\tilde{\Omega}^{rs} = 0 \quad \forall rs$

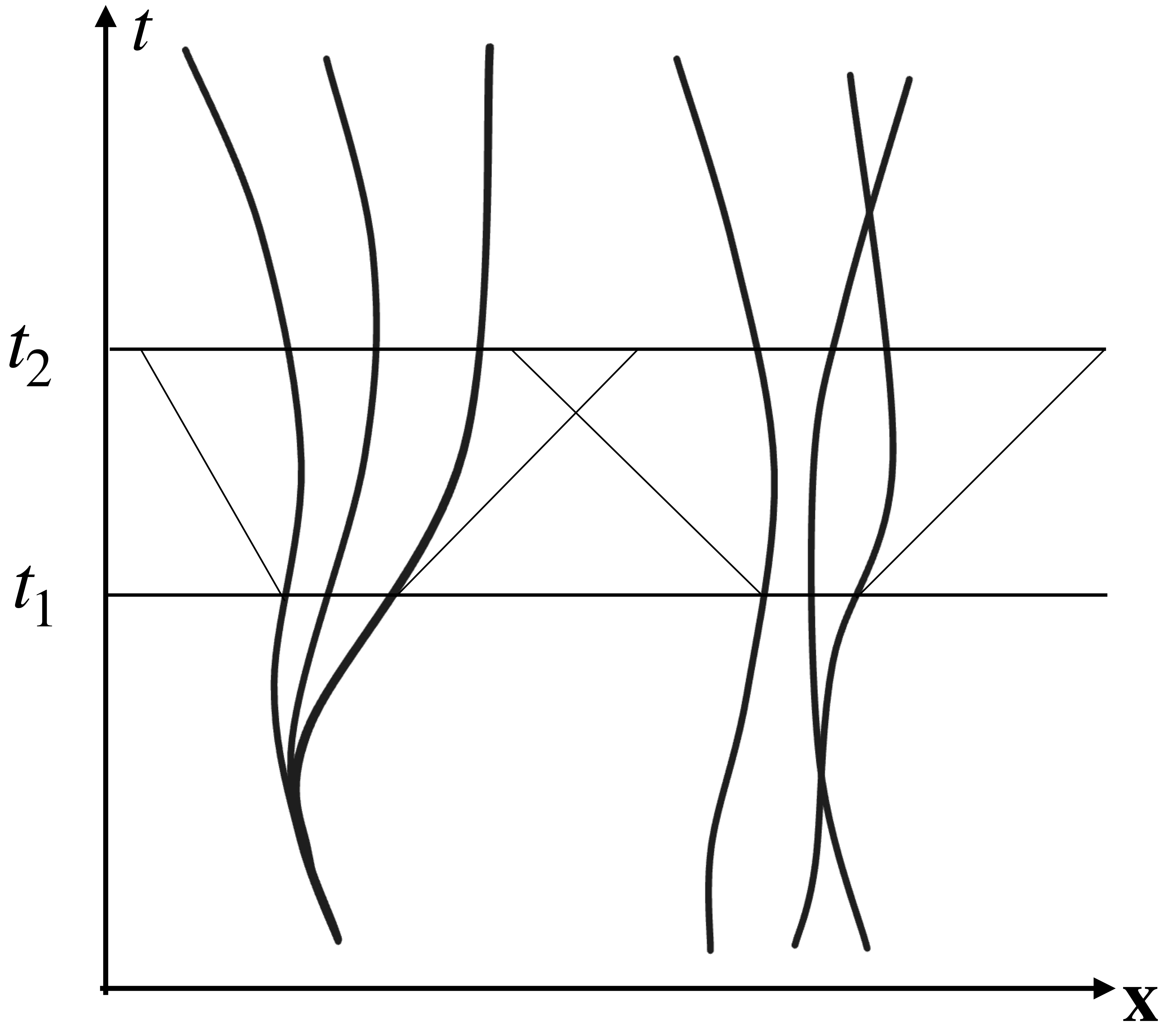


derivation

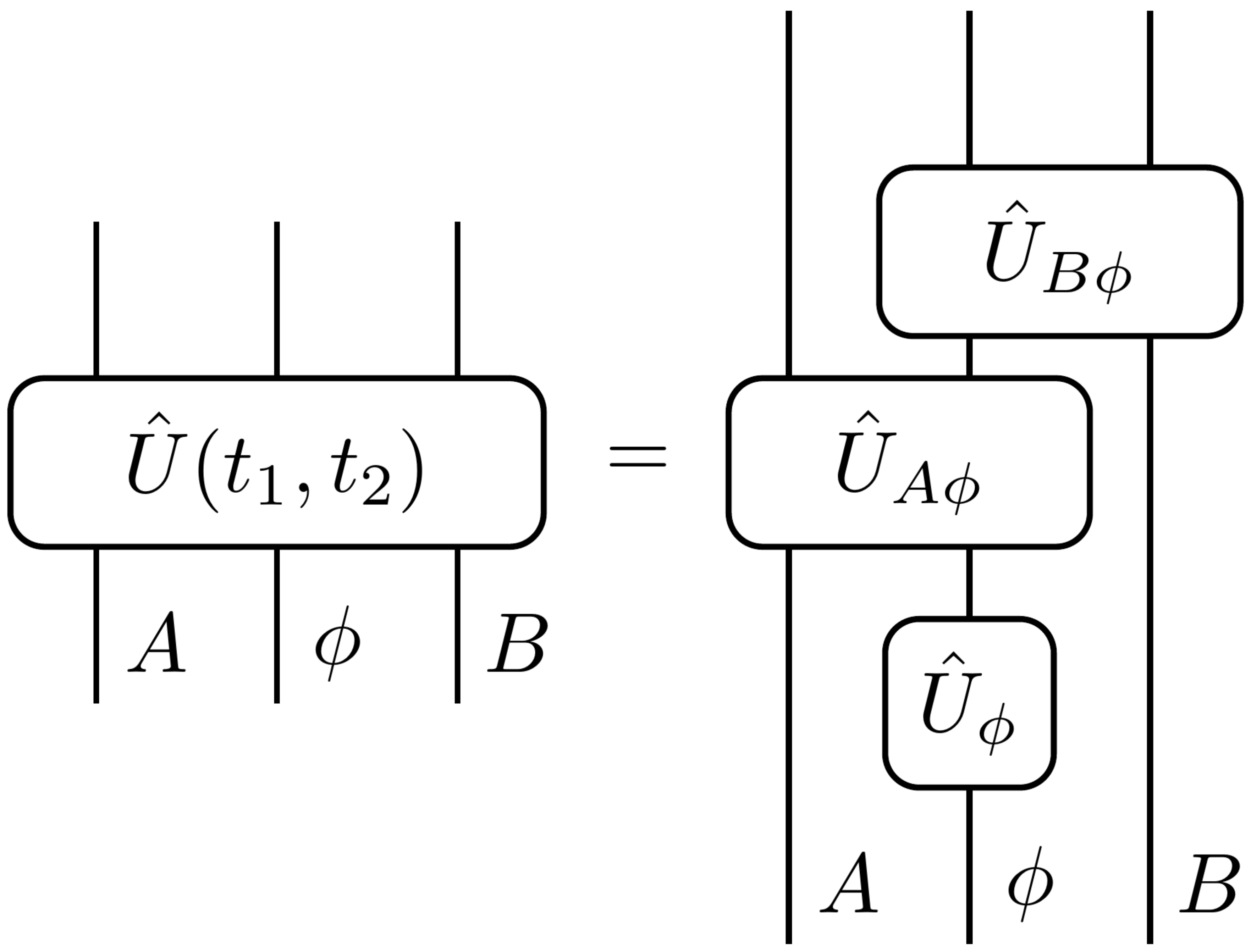
relativistic locality implies subsystem locality

if $\text{dom}\,\mu_A^r, \quad \text{dom}\,\mu_B^s$ **are spacelike** $\forall rs$

then $\hat{U}(t_1, t_2) = \hat{U}_{B\phi} \circ \hat{U}_{A\phi} \circ e^{-i\hat{H}_0(t_2 - t_1)}$



$\Longrightarrow$



Summary

- Subsystem locality and relativistic locality are related but different notions.
- Relativistic locality (via microcausality) implies subsystem locality, in a simple model, and only in a certain approximate regime.
- This is to be expected.
- Interesting intersection for RQI.

Open questions

- To what extend can this result be generalised?
  - Massless, gauge fields?
- Perhaps we need to leverage tools from AQFT.
- If result cannot be generalised, what is the impact on quantum foundations?

thank you