Probability with… **Linear Equations**

$3x + 4 = 25$	$13 - 8x = 1$	$x + 5 = 7x - 37$	$x + 11 = 6 - x$
$\frac{x}{2} - 8 = 2 + 3x$	$\frac{1}{x} = -\frac{2}{5}$	$\frac{x-2}{5} = \frac{x+2}{9}$	$3(2x - 9) = x + 8$

a) Alice chooses two equations at random.
Find the probability that both equations have integer solutions.

b) Brooke and Billie choose one of the eight equations each at random.
Find the probability their equations have the same solution.

c) Curtis chooses equations at random until he finds one with a positive solution.
He doesn't choose equations he has already solved.
Find the probability he solves exactly three equations.

Probability with… **Linear Equations**

Solutions

$3x + 4 = 25$ $x = 7$	$13 - 8x = 1$ $x = 1.5$	$x + 5 = 7x - 37$ $x = 7$	$x + 11 = 6 - x$ $x = -2.5$
$\frac{x}{2} - 8 = 2 + 3x$ $x = -4$	$\frac{1}{x} = -\frac{2}{5}$ $x = -2.5$	$\frac{x-2}{5} = \frac{x+2}{9}$ $x = 7$	$3(2x - 9) = x + 8$ $x = 7$

a) Alice chooses two equations at random. Find the probability that both equations have integer solutions.

$$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \mathbf{\frac{5}{14}}$$

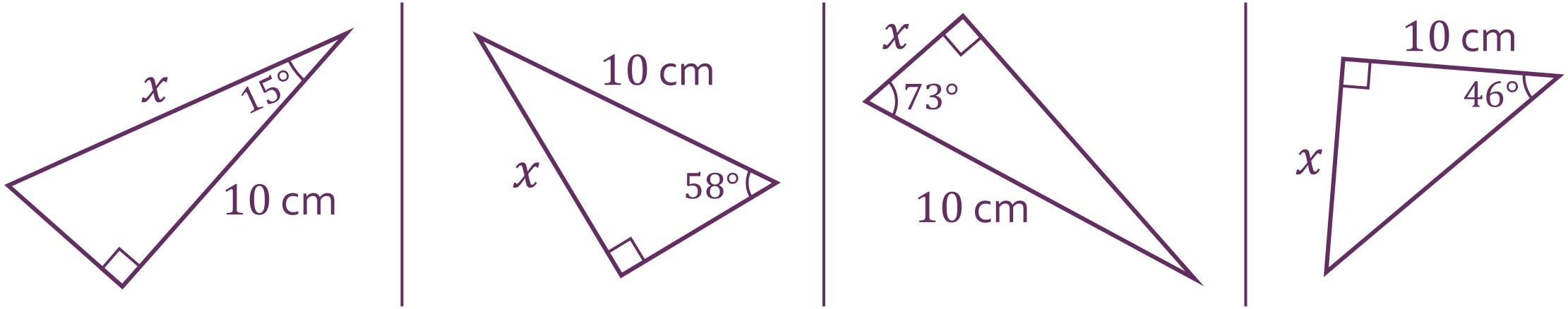
b) Brooke and Billie choose one of the eight equations each at random.
Find the probability their equations have the same solution.

$$\left(\frac{4}{8} \times \frac{4}{8}\right) + \left(\frac{2}{8} \times \frac{2}{8}\right) + \left(\frac{1}{8} \times \frac{1}{8}\right) + \left(\frac{1}{8} \times \frac{1}{8}\right) = \frac{16}{64} + \frac{4}{64} + \frac{1}{64} + \frac{1}{64} = \frac{22}{64} = \mathbf{\frac{11}{32}}$$

c) Curtis chooses equations at random until he finds one with a positive solution.
He doesn't choose equations he has already solved. Find the probability he solves exactly three equations.

$$\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} = \frac{30}{336} = \mathbf{\frac{5}{56}}$$

Probability with… **Right-Angled Trigonometry**



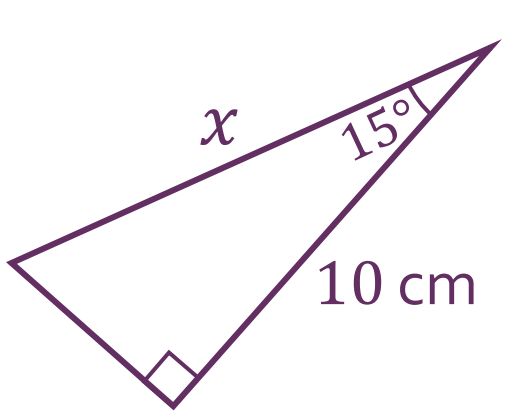
a) Ava and Aisling both choose one of the four questions at random to answer (in one step).
Find the probability that would **not** use the same trigonometric ratio.

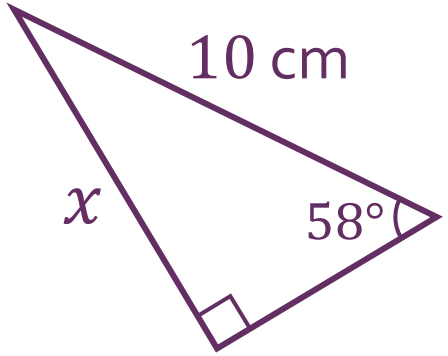
b) Baktash chooses two questions at random to answer.
Find the probability both of his answers are greater than 10 cm.

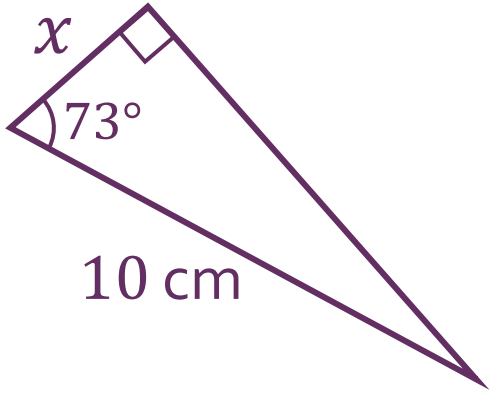
c) Casey answers all four questions, but randomly guesses each time whether to use sin, cos or tan. Find the probability he guesses correctly all four times.

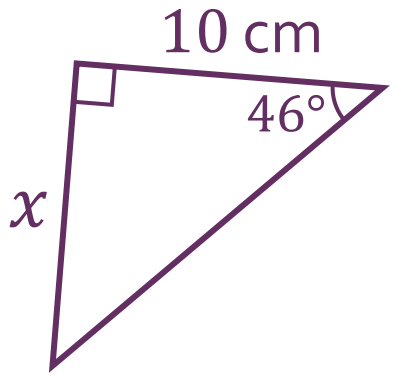
Probability with… **Right-Angled Trigonometry**

Solutions









a) Ava and Aisling both choose one of the four questions at random to answer (in one step). Find the probability that would **not** use the same trigonometric ratio.

$$1-\left(\frac{2}{4}\times\frac{2}{4}\right)-\left(\frac{1}{4}\times\frac{1}{4}\right)-\left(\frac{1}{4}\times\frac{1}{4}\right)=1-\frac{4}{16}-\frac{1}{16}-\frac{1}{16}=\frac{10}{16}=\mathbf{\frac{5}{8}}$$

b) Baktash chooses two questions at random to answer. Find the probability both of his answers are greater than 10 cm.

$$\frac{2}{4}\times\frac{1}{3}=\frac{2}{12}=\mathbf{\frac{1}{6}}$$

c) Casey answers all four questions, but randomly guesses each time whether to use sin, cos or tan. Find the probability he guesses correctly all four times.

$$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\mathbf{\frac{1}{81}}$$

Probability with… **Vectors**

$\begin{pmatrix}6\\-2\end{pmatrix}$	$\begin{pmatrix}-1\\3\end{pmatrix}$	$\begin{pmatrix}-3\\1\end{pmatrix}$	$\begin{pmatrix}-5\\0\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}2\\6\end{pmatrix}$
$\begin{pmatrix}0\\3\end{pmatrix}$	$\begin{pmatrix}3\\0\end{pmatrix}$	$\begin{pmatrix}3\\1\end{pmatrix}$	$\begin{pmatrix}9\\-3\end{pmatrix}$	$\begin{pmatrix}-6\\2\end{pmatrix}$	$\begin{pmatrix}3\\-9\end{pmatrix}$

a) Aaron and Anja choose two different vectors at random.
Find the probability their vectors are parallel.

b) Blaine chooses three different vectors at random.
Find the probability that none of the three vectors describe a horizontal movement.

c) Charlotte keeps choosing any of the 12 vectors at random until she gets one parallel to $\begin{pmatrix}3\\-1\end{pmatrix}$.
Find the probability she is successful on her fourth attempt.

Probability with… **Vectors**

Solutions

$\begin{pmatrix}6\\-2\end{pmatrix}$	$\begin{pmatrix}-1\\3\end{pmatrix}$	$\begin{pmatrix}-3\\1\end{pmatrix}$	$\begin{pmatrix}-5\\0\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}2\\6\end{pmatrix}$
$\begin{pmatrix}0\\3\end{pmatrix}$	$\begin{pmatrix}3\\0\end{pmatrix}$	$\begin{pmatrix}3\\1\end{pmatrix}$	$\begin{pmatrix}9\\-3\end{pmatrix}$	$\begin{pmatrix}-6\\2\end{pmatrix}$	$\begin{pmatrix}3\\-9\end{pmatrix}$

a) Aaron and Anja choose two different vectors at random. Find the probability their vectors are parallel.

$$\left(\frac{4}{12}\times\frac{3}{11}\right)+\left(\frac{3}{12}\times\frac{2}{11}\right)+\left(\frac{2}{12}\times\frac{1}{11}\right)=\frac{12}{132}+\frac{6}{132}+\frac{2}{132}=\frac{20}{132}=\mathbf{\frac{5}{33}}$$

b) Blaine chooses three different vectors at random.

Find the probability that none of the three vectors describe a horizontal movement.

$$\frac{9}{12}\times\frac{8}{11}\times\frac{7}{10}=\frac{504}{1320}=\mathbf{\frac{21}{55}}$$

c) Charlotte keeps choosing any of the 12 vectors at random until she gets one parallel to $\begin{pmatrix}3\\-1\end{pmatrix}$.

Find the probability she is successful on her fourth attempt.

$$\frac{8}{12}\times\frac{8}{12}\times\frac{8}{12}\times\frac{4}{12}=\frac{2048}{20736}=\mathbf{\frac{8}{81}}$$

Probability with… **Sequences**

$6, 8, 10, 12, \ldots$	$4, 40, 400, 4000, \ldots$	$1, 4, 9, 16, \ldots$	$10, 17, 24, 31, \ldots$
$100, 99, 98, \ldots$	$2, 5, 8, 11, \ldots$	$1, 1, 2, 3, 5, \ldots$	$-17, -12, -7, -2, \ldots$

a) Alia chooses two sequences at random.
Find the probability that exactly one of the sequences contains the term 38.

b) Bilal, Bonnie and Bryony all choose one of the eight sequences at random.
Find the probability that none of the chosen sequences are arithmetic.

c) Ciaran chooses sequences until he chooses one which only has even terms.
He doesn't choose sequences he has already picked.
Find the probability he chooses fewer than three sequences.

Probability with… **Sequences**

Solutions

$6, 8, 10, 12, \ldots$	$4, 40, 400, 4000, \ldots$	$1, 4, 9, 16, \ldots$	$10, 17, 24, 31, \ldots$
$100, 99, 98, \ldots$	$2, 5, 8, 11, \ldots$	$1, 1, 2, 3, 5, \ldots$	$-17, -12, -7, -2, \ldots$

a) Alia chooses two sequences at random. Find the probability that exactly one of the sequences contains the term 38.

$$\left(\frac{5}{8}\times\frac{3}{7}\right)+\left(\frac{3}{8}\times\frac{5}{7}\right)=\frac{15}{56}+\frac{15}{56}=\frac{30}{56}=\mathbf{\frac{15}{28}}$$

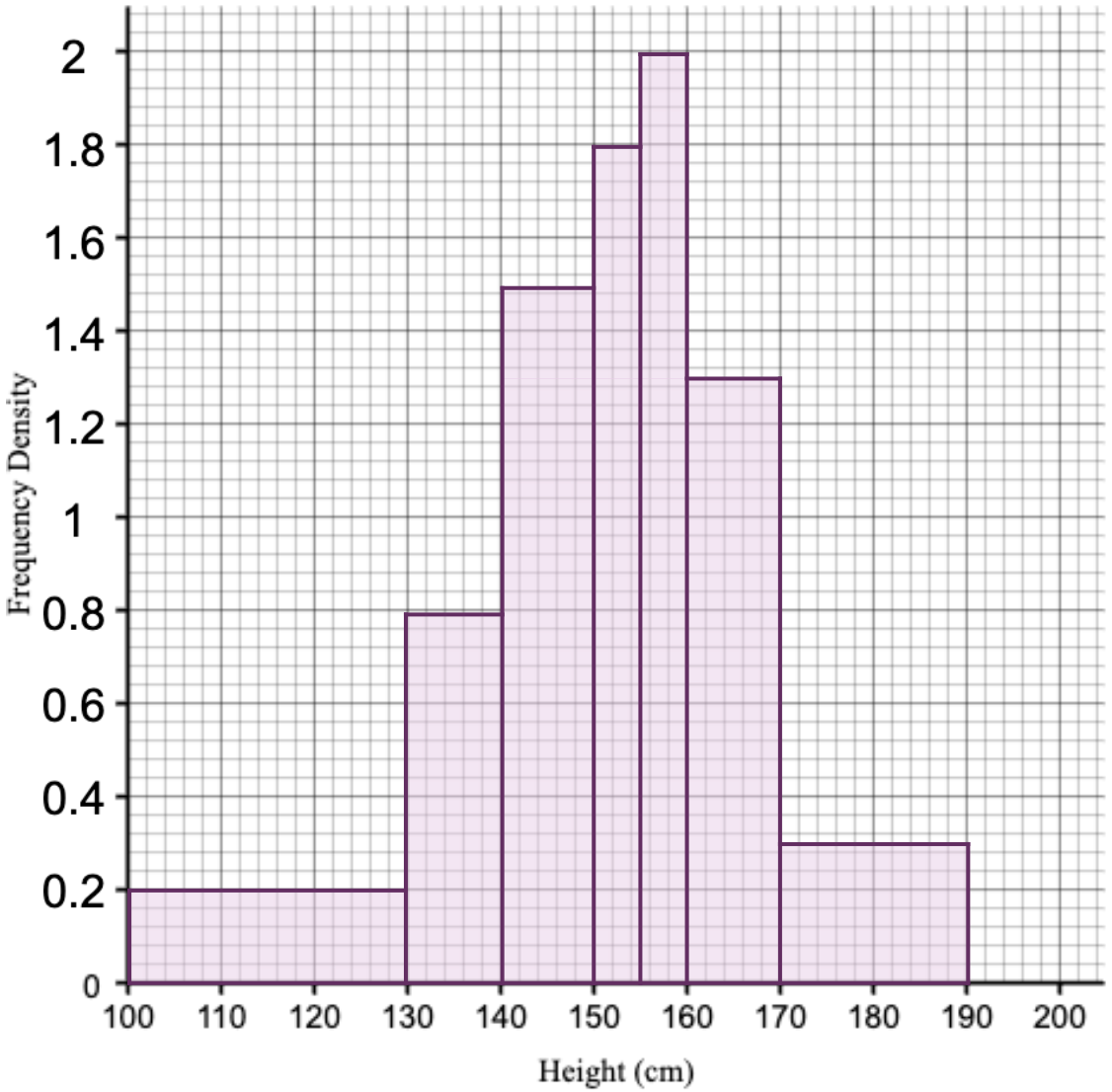
b) Bilal, Bonnie and Bryony all choose one of the eight sequences at random.
Find the probability that none of the chosen sequences are arithmetic.

$$\frac{3}{8}\times\frac{3}{8}\times\frac{3}{8}=\mathbf{\frac{27}{512}}$$

c) Ciaran chooses sequences until he chooses one which only has even terms.
He doesn't choose sequences he has already picked. Find the probability he chooses fewer than three sequences.

$$\frac{2}{8}+\frac{6}{8}\times\frac{2}{7}=\frac{2}{8}+\frac{12}{56}=\frac{14}{56}+\frac{12}{56}=\frac{26}{56}=\mathbf{\frac{13}{28}}$$

Probability with… **Histograms**



The histogram shows the heights of a group of people. Two people are chosen at random.

Find the probability that…

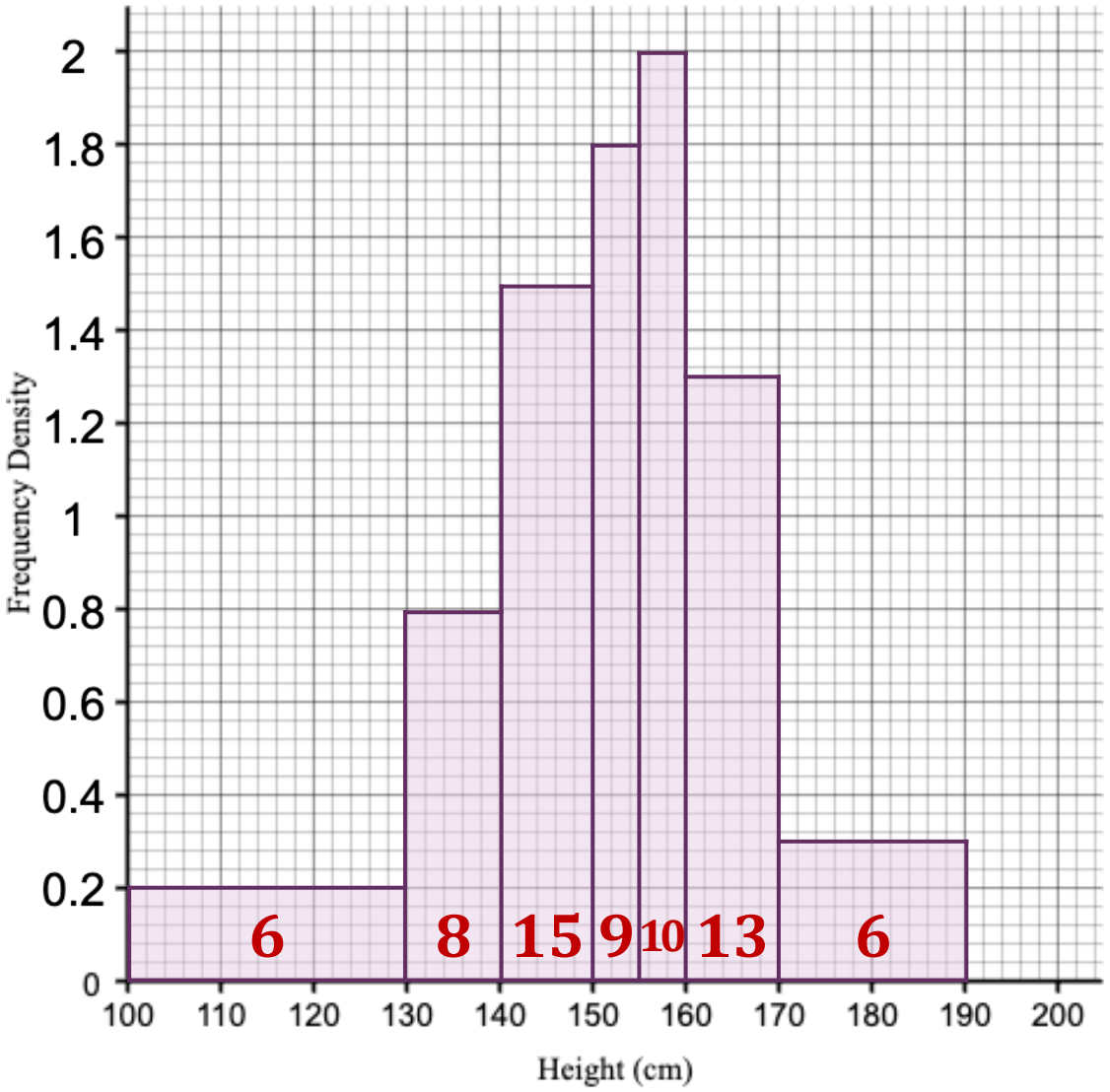
a) … both people are taller than 170 cm.

b) … one of the people is shorter than 140 cm.

c) … the two people are in the same group.

Probability with… **Histograms**

Solutions



(With thanks to mathsbot.com/printables/graphPaper)

The histogram shows the heights of a group of people. Two people are chosen at random.

Find the probability that…

a) … both people are taller than 170 cm.

$$\frac{6}{67} \times \frac{5}{66} = \mathbf{\frac{5}{737}}$$

b) … one of the people is shorter than 140 cm.

$$\frac{14}{67} \times \frac{53}{66} + \frac{53}{67} \times \frac{14}{66} = \frac{742}{4422} + \frac{742}{4422} = \frac{1484}{4422} = \mathbf{\frac{742}{2211}}$$

c) … the two people are in the same group.

$$\left(\frac{6}{67} \times \frac{5}{66}\right) + \left(\frac{8}{67} \times \frac{7}{66}\right) + \left(\frac{15}{67} \times \frac{14}{66}\right) + \left(\frac{9}{67} \times \frac{8}{66}\right) + \left(\frac{10}{67} \times \frac{9}{66}\right) + \left(\frac{13}{67} \times \frac{12}{66}\right) + \left(\frac{6}{67} \times \frac{5}{66}\right) = \mathbf{\frac{623}{4422}}$$

Probability with… **Equations of Straight Lines**

$y = 2x - 5$	$y = x + 3$	$y = -\frac{1}{2}x - 5$	$y = 3x - 13$
$y - 2x = 4$	$-x = 3y - 12$	$2y = 5x - 18$	

a) Ayyub chooses an equation at random. He then chooses one of the remaining equations.
Find the probability that both lines have the same gradient.

b) Beth and Brad both draw the graphs of one of the seven equations at random.
Find the probability that exactly one of their lines goes through the point $(8, 11)$.

c) Clara chooses two different equations at random.
Find the probability that the two lines are perpendicular.

Probability with… **Equations of Straight Lines**

Solutions

$y = 2x - 5$	$y = x + 3$	$y = -\frac{1}{2}x - 5$	$y = 3x - 13$
$y - 2x = 4$ $y = 2x + 4$	$-x = 3y - 12$ $y = -\frac{1}{3}x + 4$	$2y = 5x - 18$ $y = \frac{5}{2}x - 9$	

a) Ayyub chooses an equation at random. He then chooses one of the remaining equations.
Find the probability that both lines have the same gradient.

$\frac{2}{7} \times \frac{1}{6} = \frac{2}{42} = \mathbf{\frac{1}{21}}$

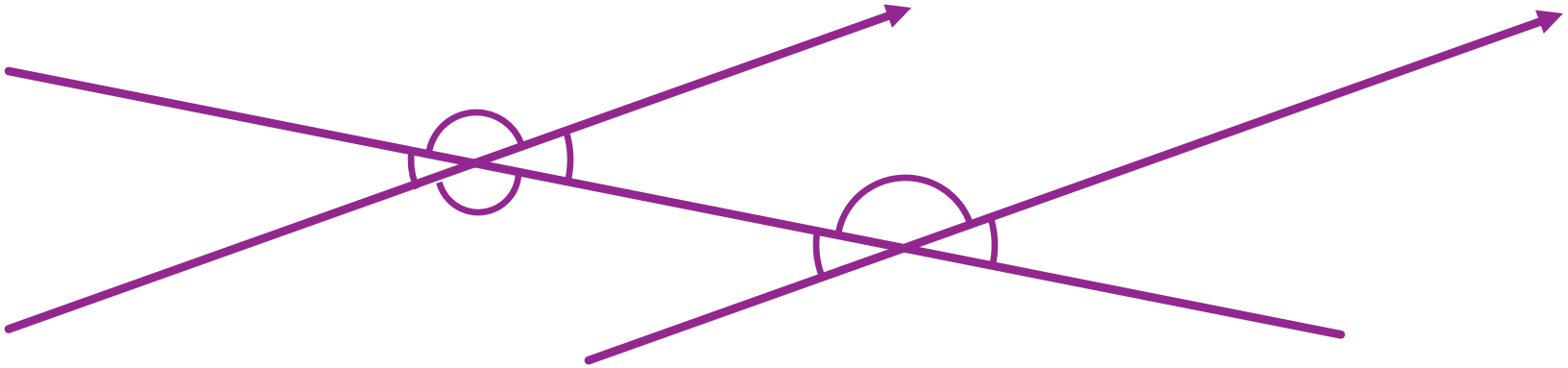
b) Beth and Brad both draw the graphs of one of the seven equations at random.
Find the probability that exactly one of their lines goes through the point $(8, 11)$.

$\left(\frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7}\right) = \frac{12}{49} + \frac{12}{49} = \mathbf{\frac{24}{49}}$

c) Clara chooses two different equations at random. Find the probability that the two lines are perpendicular.

$\left(\frac{2}{7} \times \frac{1}{6}\right) + \left(\frac{1}{7} \times \frac{2}{6}\right) + \left(\frac{1}{7} \times \frac{1}{6}\right) + \left(\frac{1}{7} \times \frac{1}{6}\right) = \frac{6}{42} = \mathbf{\frac{1}{7}}$

Probability with… **Angles in Parallel Lines**



Two of the seven marked angles are chosen at random.

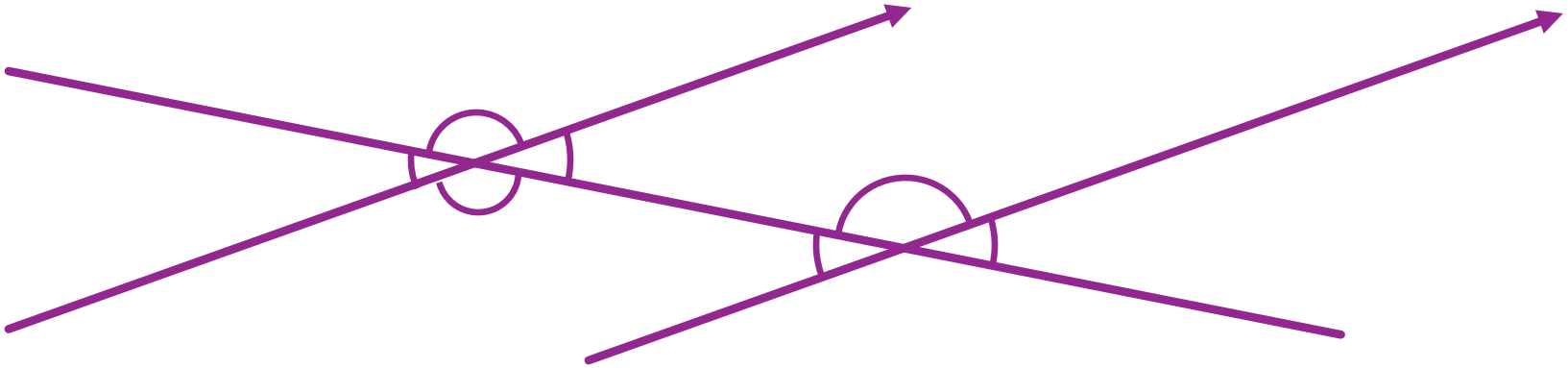
Find the probability that…

a) … the angles are equal.
b) … the angles are corresponding.
c) … the angles are co-interior.
d) … the angles are both acute.

How would your answers change if the same angle could be selected twice?

Probability with… **Angles in Parallel Lines**

Solutions



Two of the seven marked angles are chosen at random.
Find the probability that…

a) … the angles are equal.

$$\left(\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{3}{7}\times\frac{2}{6}\right)=\frac{12}{42}+\frac{6}{42}=\frac{18}{42}=\mathbf{\frac{3}{7}}$$

b) … the angles are corresponding.

$$\frac{6}{7}\times\frac{1}{6}=\frac{3}{42}=\mathbf{\frac{1}{7}}$$

c) … the angles are co-interior.

$$\frac{4}{7}\times\frac{1}{6}=\frac{4}{42}=\mathbf{\frac{2}{21}}$$

d) … the angles are both acute.

$$\frac{4}{7}\times\frac{3}{6}=\mathbf{\frac{2}{7}}$$

Probability with… **Surds**

$\sqrt{20}$	$\sqrt{70}$	$\sqrt{3}$	$\sqrt{42}$	$\sqrt{10}$
$\sqrt{75}$	$\sqrt{30}$	$\sqrt{80}$	$\sqrt{5}$	$\sqrt{45}$

a) Alex and Andrea each choose one of the ten surds at random.
Find the probability that their surds multiply together to give 10.

b) Bella chooses three different surds at random.
Find the probability that none of them simplify.

c) Courtney simplifies surds until she finds one in the form $k\sqrt{5}$, where k is an integer greater than 1. She doesn't choose surds she has already picked.
Find the probability she simplifies more than three surds.

Probability with… **Surds**

Solutions

$\sqrt{20}$	$\sqrt{70}$	$\sqrt{3}$	$\sqrt{42}$	$\sqrt{10}$
$\sqrt{75}$	$\sqrt{30}$	$\sqrt{80}$	$\sqrt{5}$	$\sqrt{45}$

a) Alex and Andrea each choose one of the ten surds at random.
Find the probability that their surds multiply together to give 10.

$$\frac{3}{10} \times \frac{1}{10} = \frac{\mathbf{3}}{\mathbf{100}}$$

b) Bella chooses three different surds at random. Find the probability that none of them simplify.

$$\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{120}{720} = \frac{\mathbf{1}}{\mathbf{6}}$$

c) Courtney simplifies surds until she finds one in the form $k\sqrt{5}$, where k is an integer greater than 1.
She doesn't choose surds she has already picked. Find the probability she simplifies more than three surds.

$$\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{210}{720} = \frac{\mathbf{7}}{\mathbf{24}}$$