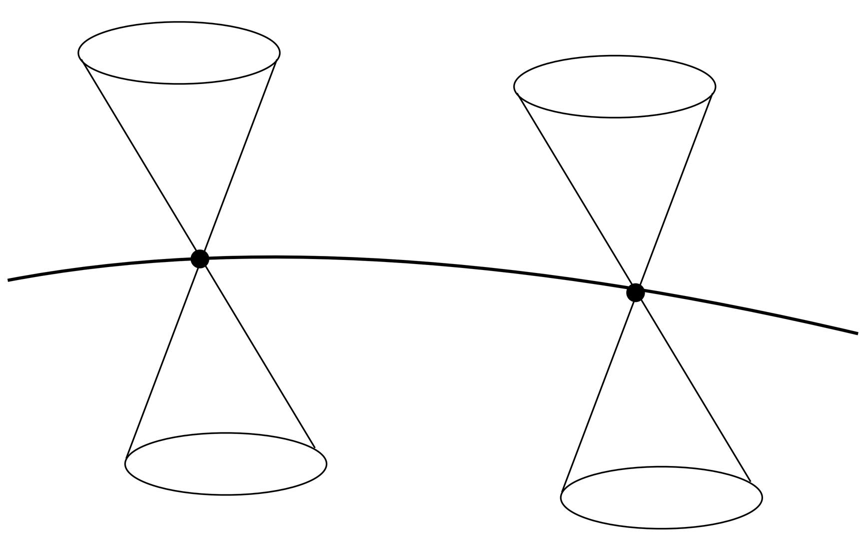
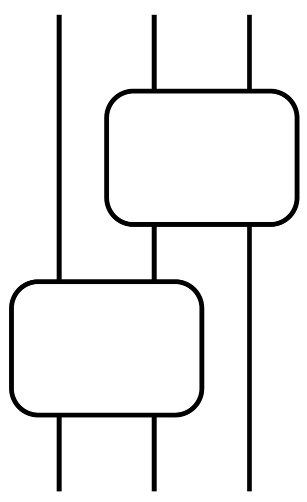
Circuit Locality from Relativistic Locality in Scalar Field Mediated Entanglement

Andrea Di Biagio, Richard Howl, Časlav Brukner, Carlo Rovelli, and Marios Christodoulou

Two notions of locality



Relativity
spacetime regions and metric
represented by manifold
information travels within lightcones



Quantum Information
subsystems and interactions
represented by circuits
information travels along wires

A knotty relationship

We normally assume that the world can be carved into **subsystems**, often using **spacetime** notions (far means separate), or even **relativistic** notions (what I do here cannot immediately influence what happens there). But there are fundamental obstructions to to this idea coming from **QM** and **QFT** (systems are just not localised in space).

The **circuit notion of locality** can only be expected to hold only in certain regimes, and only approximately because of **localisability problems in QFT**. Open problem to map the **emergence** of such widespread notion.

Here, we focus on a specific **question**:

If two particles are separately coupled to a field,

$$\hat{H} = \hat{H}_{A\phi} + \hat{H}_{B\phi}$$

do we have circuit locality of the form

$$\hat{U} = \hat{U}_{B\phi}\hat{U}_{A\phi} \text{ ?}$$

Suzuki Trotter approach:

$$e^{-i(\hat{H}_{A\phi}+\hat{H}_{B\phi})t} = \lim_{n\to\infty}\left(e^{-i\hat{H}_{A\phi}t/n}e^{-i\hat{H}_{B\phi}t/n}\right)^n$$

$\Longrightarrow$ no insight with respect to relativity

Yes
but only approximately,
in certain regimes

Scalar Field Mediated Entanglement

Setup

two quantum-controlled particles interacting with a field

$$\mathscr{H} = \left(\mathbb{C}^d \otimes L^2(\mathbb{R}^3)\right)^2 \otimes \mathscr{F}$$

$$\hat{H}(t) = \hat{H}_A(t) + \hat{H}_B(t) + \hat{H}_0 + \hat{H}_{\text{int}}$$

kinetic field term

$$\hat{H}_0 = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\omega_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}$$

$\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$

local interaction

$$\hat{H}_{\text{int}} = \int \mathrm{d}^3\mathbf{x}\,\hat{\phi}(\mathbf{x})\big(\hat{\mu}_A(\mathbf{x}) + \hat{\mu}_B(\mathbf{x})\big)$$

$\hat{\mu}_A(\mathbf{x}) = \mu(\mathbf{x} - \hat{\mathbf{x}}_A)$

quantum-controlled dynamics

$$\hat{H}_A(t) = \sum_r |r\rangle\langle r| \otimes \hat{H}^r_A(t) \qquad \hat{H}_B(t) = \sum_s |s\rangle\langle s| \otimes \hat{H}^s_B(t)$$

Parametric approximation

the qudits drive the particles, the particles drive the field

$$|\Psi(t)\rangle = \sum_{rs} c_{rs}|rs\rangle|\psi^r_A(t)\rangle|\psi^s_B(t)\rangle|\psi^{rs}_\phi(t)\rangle$$

particles: $\frac{\mathrm{d}}{\mathrm{d}t}|\psi^r_A(t)\rangle = -i\hat{H}^r_A(t)|\psi^r_A(t)\rangle$

field: $\frac{\mathrm{d}}{\mathrm{d}t}|\psi^{rs}_\phi(t)\rangle = -i\big(\hat{H}_0 + \hat{H}^{rs}_{\text{int}}(t)\big)|\psi^{rs}_\phi(t)\rangle$

$\langle\psi^r_A(t)\psi^s_B(t)|\hat{H}_{\text{int}}|\psi^r_A(t)\psi^s_B(t)\rangle$

$$\Longrightarrow \quad \hat{U} = \sum_{rs}|rs\rangle\langle rs| \otimes \hat{U}^r_A \otimes \hat{U}^s_B \otimes \hat{U}^{rs}_\phi$$

Quantum field, two semiclassical sources

($\hat{U}$ is in desired form if $\hat{U}^{rs}_\phi = \hat{U}^r_\phi\hat{U}^s_\phi$)

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}^{rs}_\phi(t) = -i\hat{H}^{rs}(t)\hat{U}^{rs}_\phi(t) \qquad \hat{H}^{rs}_{\text{int}}(t) = \int \mathrm{d}^3\mathbf{x}\,\hat{\phi}(\mathbf{x})\mu^{rs}(t,\mathbf{x})$$

$\langle\psi^r_A(t)|\hat{\mu}_A(\mathbf{x})|\psi^r_A(t)\rangle$
$+$
$\langle\psi^s_B(t)|\hat{\mu}_B(\mathbf{x})|\psi^s_B(t)\rangle$

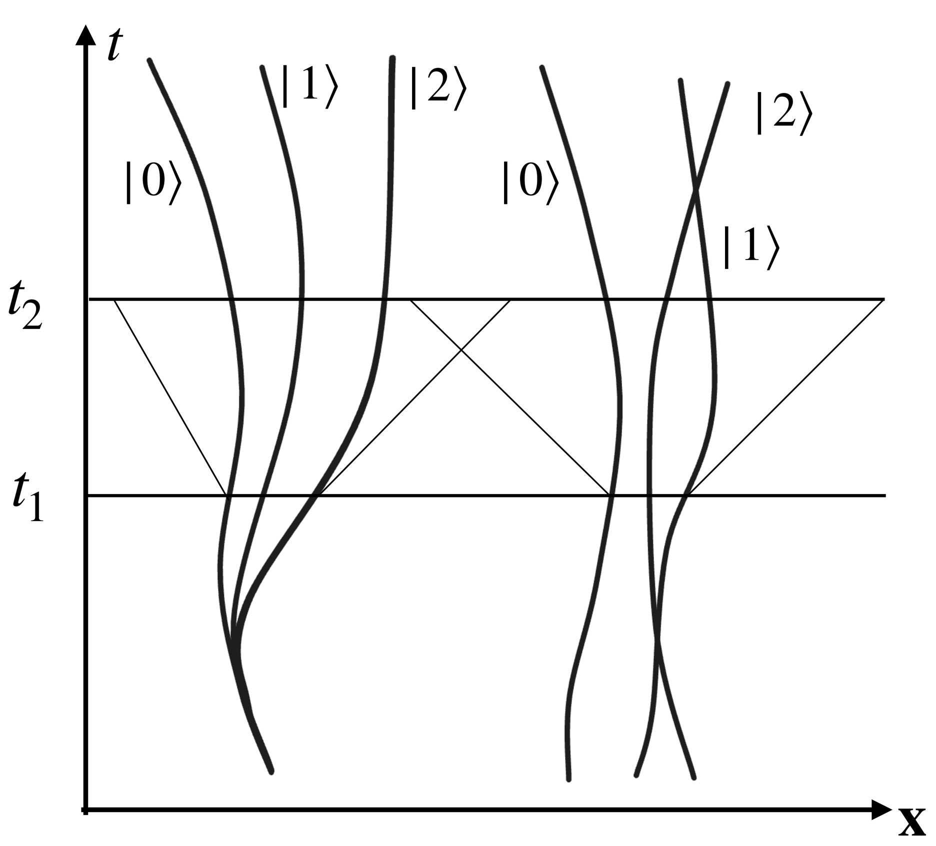
exact solution via Magnus expansion and BHC formula

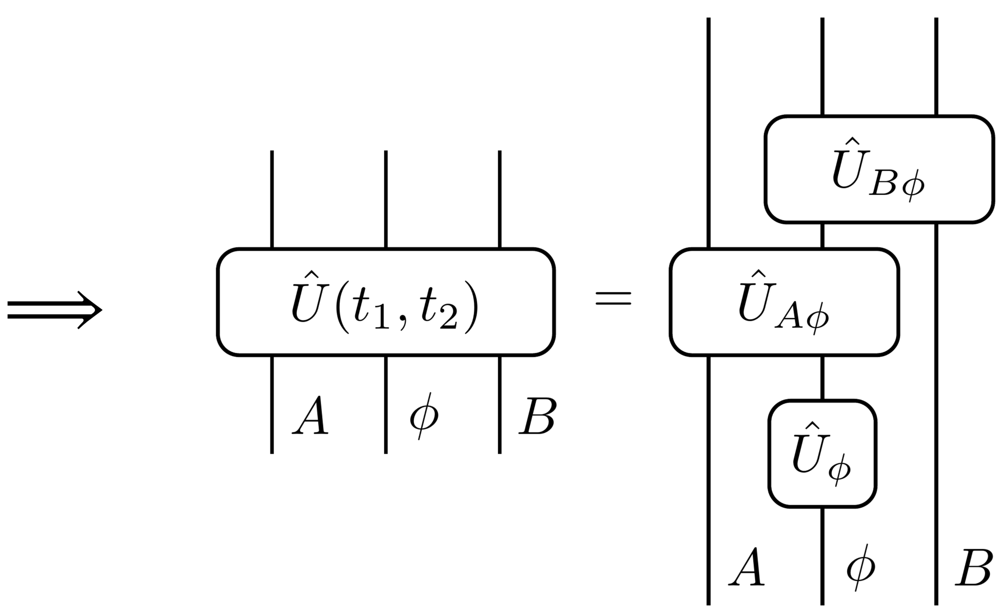
$$\hat{U}^{rs}_\phi(t_1,t_2) = e^{i\tilde{\Omega}^{rs}}\hat{D}[\mu^r_A]\hat{D}[\mu^s_B]e^{-i\hat{H}_0(t_2-t_1)}$$

a big and scary phase
(ultimately responsible for entanglement mediation)

disturbance added by the particles

free propagation





quantum-controlled superposition of localised states + spacelike separation $\Longrightarrow$ circuit-local evolution

Circuit locality from relativistic locality

If supp μ^r_A and supp μ^s_B are **spacelike separated** between t_1 and t_2, the phase

$$\tilde{\Omega}^{rs} = -i\iint_{t_1}^{t_2}\mathrm{d}t\mathrm{d}t'\iint\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{x}'\,\mu^r_A(t,\mathbf{x})\mu^s_B(t',\mathbf{x}')[\hat{\phi}_I(t,\mathbf{x}),\hat{\phi}_I(t',\mathbf{x}')]$$
$$-i\int_{t_1}^{t_2}\mathrm{d}t\int_{t_1}^{t}\mathrm{d}t'\iint\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{x}'\big(\mu^r_A(t,\mathbf{x})\mu^s_B(t',\mathbf{x}') + \mu^r_B(t,\mathbf{x})\mu^s_A(t',\mathbf{x}')\big)[\hat{\phi}_I(t,\mathbf{x}),\hat{\phi}_I(t',\mathbf{x}')]$$

vanishes thanks to **relativistic locality:**

$$[\hat{\phi}_I(t,\mathbf{x}),\hat{\phi}_I(t',\mathbf{x}')] = 0 \qquad \text{(for } (t,\mathbf{x}) \text{ and } (t',\mathbf{x}') \text{ spacelike)}$$

$$\hat{U} = \sum_{sr}\Big(|s\rangle\langle s|\hat{U}^s_B \otimes \hat{U}^s_\phi\Big)\circ\Big(|r\rangle\langle r|\hat{U}^r_A \otimes \hat{U}^r_\phi\Big)\circ e^{-i\hat{H}_0(t_2-t_1)} = \hat{U}_{B\phi}\hat{U}_{A\phi}\hat{U}_\phi$$

Quantum gravity no-go theorems

Theory-independent results about **gravity mediated entanglement** [1,2,3] and reversible gravitational interaction [4] assume **locality** in the **circuit** sense, not relativistic sense.
What are these theorems telling us if QFT does not display this kind of circuit locality?

[1] S. Bose, A. Mazumdar, GW. Morley, et. al., A Spin Entanglement Witness for Quantum Gravity arXiv:1707.06050
[2] C. Marletto, V. Vedral, Witnessing non-classicality beyond quantum theory arXiv:2003.07974
[3] T. Galley, F. Giacomini, J. Selby, A no-go theorem on the nature of the gravitational field [...] arXiv:2012.01441
[4] T. Galley, F. Giacomini, J. Selby, Any consistent coupling between classical gravity and [...]arXiv:2301.10261

Possible generalisations

Some immediate next steps:
- quantify the deviation by bounding the effect of the tails, **go beyond quantum-controlled semiclassical** states of matter (functional analysis)
- generalise to **massless gauge fields**: the displacement operators will not be defined (IR divergences)
  $\Longrightarrow$ AQFT approach (in the works)
- generalise to fields interacting with fields?

Adapted from our paper:

Relativistic locality can imply subsystem locality

arXiv:2305.05645

Scan me!